

COMPUTATION OF STRESS INTENSITY FACTORS OF AN AXISYMMETRIC INFINITE CYLINDER HAVING A TRANSVERSE CRACK

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ABSTRACT

This paper considers the problem of an axisymmetric infinite cylinder with a crack at $z = 0$. The cylinder is under the action of uniformly distributed axial tension applied at infinity and its lateral surface is free of traction. It is assumed that the material of the cylinder is linearly elastic and isotropic. Crack surfaces are free. Formulation of the boundary problem under consideration is reduced to single singular integral equation in terms of the derivative of the crack surface displacement. These equations together with the single-valuedness and equilibrium condition for the displacements around the crack is converted to a system of a linear algebraic equation which is solved numerically. Stress intensity factors are calculated and presented in graphical form.

Key Words : Crack, Singularity, Cylinder, Stress intensity factor

EKSENEL SİMETRİK ÇATLAK İÇEREN SONSUZ SİLİNDİRİN GERİLME ŞİDDETİ FAKTÖRÜNÜN HESAPLANMASI

ÖZET

Bu çalışmada çekmeye maruz $z = 0$ düzleminde çatlak içeren sonsuz ekselel simetrik silindir incelenmektedir. Silindir düzgün yayılı p_0 şiddetinde ekselel çekmeye maruzdur. Silindirin yanıl yüzeylerinde gerilmeler sıfırdır. Malzemenin lineer elastik ve izotropik olduđu varsayılmaktadır. Çatlak yüzeyleri serbesttir. Problem çatlak yüzey deplasman türevi cinsinden tek tekil integral denkleme indirgenmektedir. Boyutsuzlaştırılan denklem takımı single-valuedness ve denge durum denklemleri ilave edilerek lineer denklem takımlarına indirgenerek sayısal çözümü yapılmıştır. Gerilme şiddeti faktörü hesaplanmış ve sonuçlar grafiklerle sunulmuştur.

Anahtar Kelimeler : Çatlak, Tekillik, Silindir, Gerilme şiddeti faktörü

1. INTRODUCTION

Fracture mechanics is based on the principle that all materials contain initial defects in the form of cracks, voids or inclusions that can effect the load carrying capacity of engineering structures. The engineering field of fracture mechanics was established to develop a basic understanding of crack propagation problem. Cracks will propagate under service loading and finally could lead to a

complete failure of the structure. Fracture occurs when either the toughness of the material is exceeded or the remaining ligament yields. In other words, fracture toughness expresses the ability of the material to resist a fracture in the presence of the cracks. The stress intensity factor K , which defines the amplitude of the crack tip singularity, incorporates both geometrical terms and the stress level. Many levels of stress depending on the

magnitude of load applied and size of the component.

Various authors have considered the problem of semi-infinite and finite solid circular cylinder with the curved surface free of tractions and prescribed displacements or tractions at the plane end. The classical theory of elasticity equations are solved in terms of unknown function that is shown to be the solution of Fredholm integral equation of second kind. Erdogan and Gupta (1973) developed the pair of Gauss-Chebyshev integration formulas for singular integrals. By using these formulas a simple numerical method for solving a system of singular integral equation is described. The integral equation is replaced by a system of linear algebraic equations and this system is solved to calculate the stress intensity factor and stresses. Integral equation approach is taken by Gupta in solving the problem of a semi-infinite strip (1973) and semi-infinite cylinder (1975) under uniform tension at one end and fixed at the other. Integral transform technique is used to provide an exact formulation of the problem in terms of singular integral equations. Stress singularity at the strip corner is obtained from the singular integral equation, which is solved numerically. The singular integral equation obtained from the formulation of the problem must be such that the method of Muskhelishvili (1953) can be used. Nied and Erdoğan (1983) considered the elasticity problem for a long circular cylinder containing an axisymmetric circumferential crack subjected to a general nonaxisymmetric external loads. The axisymmetric contact problem for a semi-infinite cylinder and a half space is considered by Geçit (1986). The problem is reduced to a system of singular integral equations of the second kind using the transform technique. The problem of a semi-infinite strip containing a transverse crack is considered by (Geçit, 1988).

In the present paper, the axisymmetric infinite elastic solid circular cylinder subjected to axial tension with lateral surface free of traction and having a transverse crack at $z = 0$ plane is considered. The objective of this work is to investigate the stress intensity factors for the ring-shaped crack located on $z = 0$ plane symmetrically with arbitrary (but equal) length. Material of the cylinder is assumed to be linearly elastic and isotropic. The solution of the actual problem can be obtained by superposition of solutions for the following two problems: (1) Uniform solution of an infinite cylinder subjected to uniform tension only and (2) The problem containing a ring-shaped transverse crack of arbitrary width $0 < (b - a) < A$ at $z = 0$. Loading in the second problem is the negative of the stresses and the

displacements resulting from the first problem at locations of the crack. Solution of the first problem is obtained by applying the elasticity theory to it and called the uniform solution. The second problem is called the perturbation problem. In this study, the auxiliary solutions for the necessary subproblems can be derived by the application of Fourier and Hankel transform techniques. (Sneddon, 1951; Erdelyi, 1953) have obtained the general expressions for displacements and stress components on Navier equations. Applying the boundary conditions, the problem is reduced to a system of three singular integral equations in terms of crack surface displacement derivative. By using Gauss quadrature formulas, this singular integral equation is converted to a linear algebraic equation that is solved numerically.

2. FORMULATION OF THE RING-SHAPED CRACK PROBLEM

Consider the axisymmetric, linearly elastic, isotropic infinite cylinder with radius A shown in Figure 1. Both ends of this infinite cylinder are under the action of a uniformly distributed axial tension of intensity p_0 and the lateral surface is free of traction.

Solution for the infinite cylinder having a crack will be obtained by superposing the solutions of (i) the infinite cylinder subjected to uniformly distributed axial tension intensity p_0 at infinity, and (ii) perturbation problem for the infinite cylinder having a ring shaped transverse crack. (Figure 2).

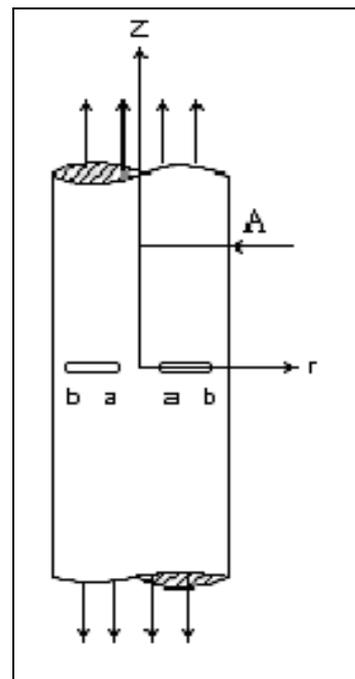


Figure 1. Geometry of the problem and loading conditions

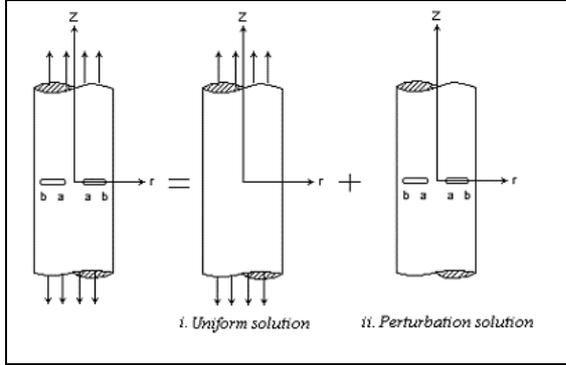


Figure 2. Superposition scheme for the general problem

For part (i) the partial differential equations are used for uniform solution that are obtained from Eqn's 1 and 2 respectively. These equations become uncoupled ordinary differential equations because of only one dependent displacement u in r -direction and one dependent w in z -direction. For part (ii) solution is obtained for the infinite cylinder contains a ring-shaped transverse crack of arbitrary length $0 < (b-a) < A$ at $z=0$ which is the plane of symmetry. Therefore, for axisymmetric elasticity problem under consideration the Navier equations may be written in the following form:

$$2\left(\frac{\partial^2 u}{\partial r \partial z} + \frac{1}{r} \frac{\partial u}{\partial z}\right) + (\kappa - 1)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r}\right) + (\kappa + 1)\frac{\partial^2 w}{\partial z^2} = 0 \quad (1)$$

$$(\kappa + 1)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + (\kappa - 1)\frac{\partial^2 u}{\partial z^2} + 2\frac{\partial^2 w}{\partial r \partial z} = 0 \quad (2)$$

where u and w are the r and z components of the displacement vector and $\kappa = 3 - 4\nu$, ν being the

$$u_2(r, z) = \frac{1}{\kappa + 1} \int_0^\infty [\kappa - 1 - 2\rho z] M(\rho) e^{-\rho z} J_1(\rho r) d\rho + \frac{2}{\pi} \int_0^\infty \left[-\frac{1}{2} c_1 I_1(\alpha r) + c_2 \alpha r I_0(\alpha r) \right] \cos \alpha z d\alpha \quad (4a)$$

$$w_2(r, z) = \frac{1}{\kappa + 1} \int_0^\infty [-\kappa - 1 - 2\rho z] M(\rho) e^{-\rho z} J_0(\rho r) d\rho + \frac{2}{\pi} \int_0^\infty \left\{ \frac{1}{2} c_1 I_0(\alpha r) - c_2 [(\kappa - 1) I_0(\alpha r) + \alpha r I_1(\alpha r)] \right\} \sin \alpha z d\alpha \quad (4b)$$

Poisson's ratio. The relevant stress components can be written as

$$\begin{aligned} \sigma_r &= \frac{\mu}{\kappa - 1} \left[(\kappa + 1) \frac{\partial u}{\partial r} + (3 - \kappa) \left(\frac{u}{r} + \frac{\partial w}{\partial z} \right) \right], \\ \sigma_z &= \frac{\mu}{\kappa - 1} \left[(\kappa + 1) \frac{\partial w}{\partial z} + (3 - \kappa) \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) \right], \\ \sigma_\theta &= \frac{\mu}{\kappa - 1} \left[(\kappa - 1) \frac{u}{r} + (3 - \kappa) \left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} \right) \right], \\ \tau_{rz} &= \mu \left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right], \end{aligned} \quad (3 a, d)$$

Where μ is the shear modulus. As shown in Figure 3 the perturbation solution of the problem must be obtained by the sum of the expressions for the displacement and the stress components of the two subproblem (a) and (b). $z = 0$ is the plane of symmetry, the problem is considered in the region $0 \leq z \leq \infty$. The obtained expressions of displacements u_2 and w_2 are in the form :

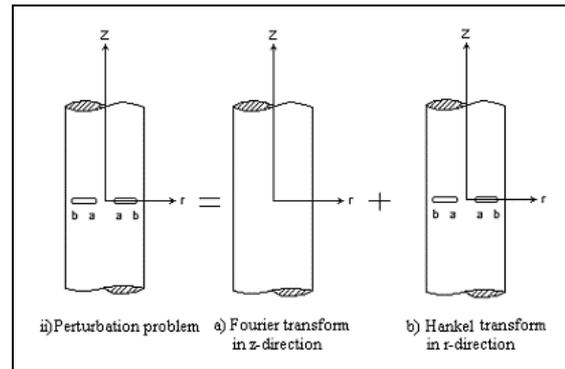


Figure 3. Superposition scheme for perturbation problem

Similarly, the obtained expressions for stresses are,

$$\begin{aligned} \sigma_{r_2} &= \frac{2\mu}{\kappa+1} \int_0^\infty \left\{ [-2\rho z - (\kappa-1)] \frac{1}{r} J_1(\rho r) + [2(1-\rho z)] \rho J_0(\rho r) \right\} M(\rho) e^{-\rho z} d\rho \\ &\quad + \frac{2\mu}{\pi} \int_0^\infty \left\{ c_1 \left[-\alpha I_0(\alpha r) + \frac{1}{r} I_1(\alpha r) \right] + c_2 \left[(\kappa-1)\alpha I_0(\alpha r) + 2\alpha^2 r I_1(\alpha r) \right] \right\} \cos \alpha z d\alpha \\ \tau_{rz_2} &= \frac{4\mu}{\kappa+1} \int_0^\infty \rho z M(\rho) e^{-\rho z} \rho J_1(\rho r) d\rho + \frac{2\mu}{\pi} \int_0^\infty \left\{ c_1 \alpha I_1(\alpha r) - c_2 \left[(\kappa+1)\alpha I_1(\alpha r) + 2r\alpha^2 I_0(\alpha r) \right] \right\} \sin \alpha z d\alpha \\ \sigma_{z_2} &= \frac{4\mu}{\kappa+1} \int_0^\infty [\rho z + 1] M(\rho) e^{-\rho z} \rho J_0(\rho r) d\rho \\ &\quad + \frac{2\mu}{\pi(\kappa+1)} \int_0^\infty \left\{ (\kappa-1)\alpha I_0(\alpha r) c_1 - 2(\kappa-1)^2 r I_1(\alpha r) c_2 + 2(3-\kappa)\alpha I_0(\alpha r) c_2 - (\kappa+1)^2 I_0(\alpha r) c_2 \right\} \cos \alpha z d\alpha \end{aligned} \tag{5a-c}$$

Where J_0 and J_1 are Bessel functions of the first kind of order zero and one, respectively, and I_0, I_1 are modified Bessel functions of the first kind of order zero and one.

$$M(\rho) = \int_a^b m(t) t J_1(\rho t) dt, \tag{6}$$

and c_1, c_2 are unknown quantities. $m(t)$ is the derivative of the crack surface displacement. c_1 and c_2 can be determined by the use of the stress boundary conditions:

$$\begin{aligned} \sigma_r(A, z) &= 0 \\ \tau_{rz}(A, z) &= 0 \end{aligned} \tag{7a, b}$$

Then,

$$\begin{aligned} c_1 &= \left\{ E_1 \left[(\kappa-1)\lambda I_0(\lambda A) + 2\lambda^2 A I_1(\lambda A) \right] - E_2 \left[-(\kappa+1)\lambda I_1(\lambda A) - 2\lambda^2 I_0(\lambda A) A^2 \right] \right. \\ &\quad \left. / \left[2\lambda^3 A^3 \left[I_1^2(\lambda A) - I_0^2(\lambda A) \right] + (\kappa+1)\lambda A I_1^2(\lambda A) \right] \right\} \\ c_2 &= \left\{ \lambda A^2 I_1(\lambda A) E_2 - \left[-\lambda I_0(\lambda A) + \frac{1}{A} I_1(\lambda A) \right] E_1 \right\} / \left[2\lambda^3 A^3 \left[I_1^2(\lambda A) - I_0^2(\lambda A) \right] + (\kappa+1)\lambda A I_1^2(\lambda A) \right] \end{aligned} \tag{8a, b}$$

Where $E_1,$ and E_2 are given in Appendix. Now for the remaining one unknown, the following boundary condition on the crack must be used :

$$\begin{aligned} \sigma_z(r, 0) &= -p_0, \\ a < r < b \end{aligned} \tag{9}$$

Where p_0 is the uniform axial tension. The strain ϵ_0 can be given as:

$$\epsilon_0 = \frac{(\kappa-3)p_0}{(7-\kappa)2\mu} \tag{10}$$

Note that the boundary condition is in terms of stress type condition (9). Therefore, by this replacement for the perturbation problem, also some divergent integral is disregarded and the following integral equations are obtained with kernels having Cauchy-type singularity.

$$\frac{2\mu}{\pi(\kappa+1)} \int_a^b \left[\frac{2}{t-r} + 2H_1(r, t) + tN_{11}(r, t) \right] m(t) dt = -p_0 \tag{11}$$

Where,

$$H_1(r, t) = \frac{m_1(r, t) - 1}{t - r} \quad (12)$$

$$m_1(r, t) = \left[\begin{array}{c} \frac{2(t-r)}{r} K\left(\frac{t}{r}\right) + \frac{2r}{t+r} E\left(\frac{t}{r}\right) \\ \frac{2t}{t+r} E\left(\frac{r}{t}\right) \end{array} \right] \quad (13)$$

K and E being the complete elliptic integrals of the first and second kinds, respectively. $N_{11}(r, t)$, is the kernel and defined in Equation (15). The system of singular integral equation, Eq. (11) must be solved subject to the following single-valuedness and equilibrium condition

$$\int_a^b m(t) dt = 0 \quad (14)$$

The dominant part of the integral equation (11) has

- i) Cauchy have with Cauchy-type singularity at $t = r$,
- ii) the kernels H_1 , have only logarithmic singularity,
- iii) among the Fredholm kernels, N_{11} have singular terms when $t = A$ and $r = \pm A$.

$$N_{11}(r, t) = \int_0^\infty L_{11}(r, t, \lambda) d\lambda \quad (15)$$

Where L_{11} is the integrands and contain the Bessel function. The singularity at zero may easily be removed when examining the behavior of integrand L_{11} for $\lambda \rightarrow 0$. It can be shown that the integrand of the kernel vanish and is bounded everywhere except for $\lambda = 0$, by examining the behavior of integrand for $\lambda \rightarrow \infty$. The singular term can be separated by studying the asymptotic behavior of the integral given by Eq. (15).

$$N_{11s}(r, t) = \int_0^\infty L_{11s}(r, t, \lambda) d\lambda \quad (16)$$

$$L_{11s}(r, t, \lambda) = \lim_{\lambda \rightarrow \infty} L_{11}(r, t, \lambda)$$

Where

$$L_{11s} = \frac{e^{-\lambda(2A-r-t)}}{\sqrt{rt}} [-4(A-r)(A-t)\lambda^2 + 2(A-r)\lambda + 6(A-t)\lambda - 4] \quad (17)$$

The bounded parts of the kernels can be expressed as

$$N_{11b}(r, t) = \int_0^\infty [L_{11}(r, t, \lambda) - L_{11s}(r, t, \lambda)] d\lambda \quad (18)$$

Corresponding singular kernels N_{11s} can be obtained by integrating L_{11s} as

$$N_{11s}(r, t) = \left[-2 + 12(A-r) \frac{d}{dr} - 4(A-r)^2 \frac{d^2}{dr^2} \right] \left[\frac{1}{r+t-2A} \right] \quad (19)$$

Together with $1/(t-r)$, N_{11} gives generalized Cauchy kernel. After somewhat lengthy manipulations, it can be shown that the solution of Eq.(11) is,

$$m(t) = \frac{M^*(t)}{[(t-a)(b-t)]^\gamma}, \quad -1 < \text{Re}(\gamma) < 0 \quad (20)$$

Where $M^*(t)$ are Hölder-continuous function in the intervals $[a, b]$. The characteristic equation for constant γ is

$$\cot \pi \gamma = 0 \quad (21)$$

Which gives $\gamma = 1/2$ at the tip of the crack ($r \rightarrow a, r \rightarrow b$). To solve the system of one singular integral equation (11) with single valuedness and equilibrium conditions, the normalization will be done. By using the Gauss-Lobatto integration formula, equation (11) can be replaced by a system of linear algebraic equation. From the viewpoint of fracture, the stress intensity factors at the tip of the crack are given below.

$$k_{1a} = \frac{4\mu}{\kappa + 1} \sqrt{2} \lim_{r \rightarrow a} \sqrt{r-a} \frac{M(a)}{\sqrt{b-a} \sqrt{r-a}} \quad (22a, b)$$

$$k_{1b} = -\frac{4\mu}{\kappa + 1} \sqrt{2} \lim_{r \rightarrow b} \frac{M(b)}{\sqrt{b-a} \sqrt{r-b}}$$

The normalized stress intensity factors are obtained as

$$\bar{k}_{1a} = \frac{k_{1a}}{P_0 \sqrt{\frac{b-a}{2}}} = m(-1), \quad \bar{k}_{1b} = \frac{k_{1b}}{P_0 \sqrt{\frac{b-a}{2}}} = -m(1) \quad (23a, b)$$

3. NUMERICAL RESULTS

The system of singular integral equation with single valuedness and equilibrium condition will be normalized and the dimensionless variables for crack given in Appendix are introduced. The integral techniques are used to evaluate the normalized integrals. The results were calculated for the normalized region $[1, -1]$, and a/A , b/A , L/A have been taken as the independent variables. Some of the numerical results are given in Table 1-3 and shown in Figure 3-5. The normalized stress intensity

factors k_{1a}^-, k_{1b}^- are obtained for an infinite cylinder having a ring-shaped crack with uniform tension intensity p_0 . Figure 3 shows the variation of the normalized stress intensity factors k_{1a}^-, k_{1b}^- with $(b-a)/A$ for various geometries when $\nu = 0.3$. It is observed that the normalized normal stress intensity factors increase when crack length increases.

Table 1. Variation of Normalized Stress Intensity Factors With Poisson's Ratio ν

(b-a)/A	$\nu = 0.4$		$\nu = 0.3$		$\nu = 0.2$	
	k_{1a}^-	k_{1b}^-	k_{1a}^-	k_{1b}^-	k_{1a}^-	k_{1b}^-
0.4	1.2551	1.0055	1.2551	1.0055	1.2551	1.0055
0.6	1.6185	1.1009	1.6185	1.1009	1.6185	1.1009
0.8	2.6281	1.3600	2.6281	1.3600	2.6281	1.3600

Table 2. Comparison of the Obtained Results (for same ν) With The Stress Intensity Factors For A Symmetric Crack in a Thick-Walled Cylinder Subjected to Axial Tension (Nied and Erdoğan, 1983)

CRACK LENGTH, (b-a)/A		Present Study		Nied and Erdoğan, (1983)	
a/A	b/A	k_{1a}	k_{1b}	$k_{(c)}$	$k_{(d)}$
0.505	0.595	1.024	0.984	1.028	0.985

Table 3. Variation of k_{1a}^- and k_{1b}^- With Crack Location

(b-a)/A	a+b>1.0A Closer to the lateral surface		a+b=1.0A Centered crack		a+b<1.0A Closer to the center of the cylinder	
	k_{1a}^-	k_{1b}^-	k_{1a}^-	k_{1b}^-	k_{1a}^-	k_{1b}^-
0.2	1.1042	1.0694	1.0821	0.9777	1.2803	0.9416
0.4	1.2617	1.1653	1.2551	1.0055	1.5779	0.9627
0.6	1.5574	1.2564	1.6185	1.1009	1.9529	1.0412
0.8	2.4613	2.1524	2.6281	1.3599	3.3882	1.2471

It is observed the normalized stress intensity factors k_{1a}^-, k_{1b}^- are independent of length L/A . However, k_{1a}^-, k_{1b}^- increase with increasing crack lengths. From Table 1 it is seen that there are no mean difference in stress intensity factors k_{1a}^- and k_{1b}^- for different ν values. For special case $(d-c)/(b-a) = 0.1$, $((d-c)/b = 0.09$ and $c/d = 0.505$ the stress intensity factors for a symmetric crack in a thick walled cylinder subjected to axial tension Nied and Erdoğan, (1983) are given together with the results

of this study when $a/A = 0.505$ and $b/A = 0.595$ in Table 2.

Table 3 and Figure 4-5 show comparisons for cracks and inclusions located closer to the center, and closer

to the lateral surface. If

- i) $a+b > 1.0A$ (closer to the lateral surface)
- ii) $a+b < 1.0A$ (closer to the center)

Note that, if the case i and ii compared with centered solution, the normalized stress intensity factors k_{1a}

for case i are less than these for the centered solution, but for case ii they are greater. The same comparison for the normalized stress intensity factors k_{1b} for case i are greater than the centered solution and case ii.

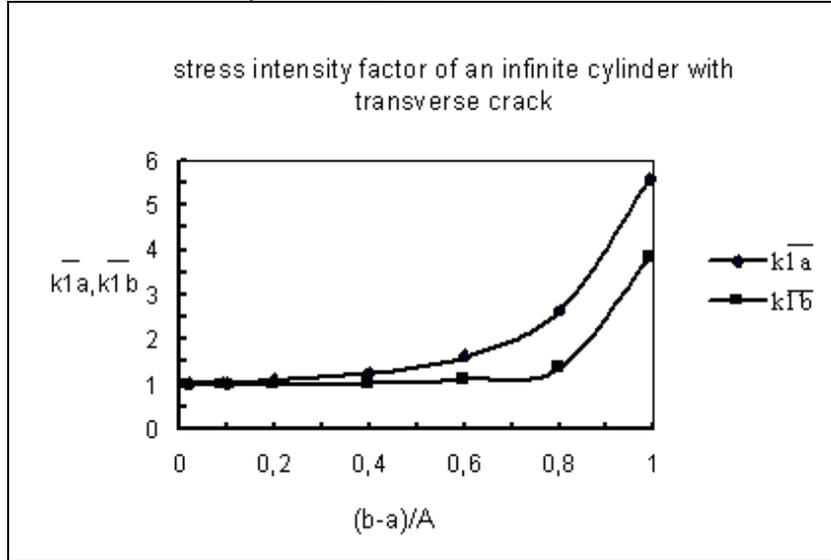


Figure 4. Variation of the Normalized Stress Intensity Factors k_{1a}, k_{1b} with $(b-a)/A$ when $\nu = 0.3$. ($a + b = 1.0A$)

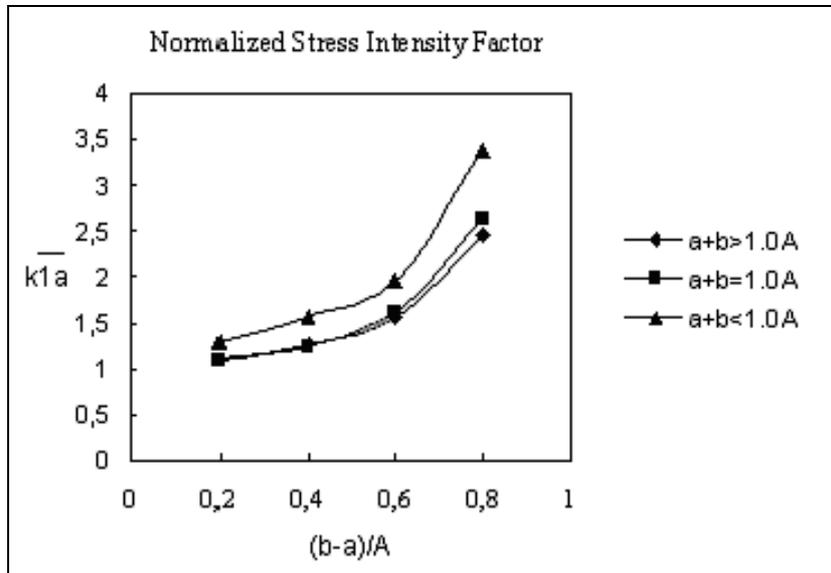


Figure 5. Variation of the k_{1a} for different $a+b$ values with $(b-a)/A$ when $\nu = 0.3$.

4. CONCLUSIONS

In this paper it has been observed that the normalized stress intensity factors k_{1a}, k_{1b} at the crack tips increase with increasing crack length. As crack approaches to the lateral surface, considerable increase occurs in the stress intensity factors. It is

concluded that the stress intensity factors depend on the material properties as well as material geometry.

$$E_1 = -\frac{8}{\kappa+1} \int_a^b m(t) dt \left[\frac{\lambda^2}{2} t K_1(\lambda A) I_0(\lambda t) - \frac{\lambda^2 A}{2} K_0(\lambda A) I_1(\lambda t) \right]$$

5. APPENDIX

Expressions for E_1 and E_2 can be put into the following form in terms of unknown functions $m(t)$:

$$E_2 = -\frac{2}{\kappa+1} \int_a^b m(t) dt \left\{ -2\lambda K_0(\lambda A) I_1(\lambda t) + \frac{2\lambda t}{A} K_1(\lambda A) I_0(\lambda t) - \frac{1}{A} \left[(\kappa+1) + 2\lambda^2 A^2 \right] K_1(\lambda A) I_1(\lambda t) + 2\lambda^2 t K_0(\lambda A) I_0(\lambda t) \right\}$$

The following dimensionless variables are introduced for crack:

$$r = \frac{b-a}{2} \xi + \frac{b+a}{2} \quad a < r < b; -1 < \xi < 1$$

$$t = \frac{b-a}{2} \tau + \frac{b+a}{2} \quad a < t < b; -1 < \tau < 1$$

6. NOTATION

- A : Radius of the cylinder
a, b : Inner and outer radii of the crack
E : Young's Modulus of Elasticity
 ρ : Hankel transform variable
 $m(r)$: Crack surface displacement derivative
 k_{ia}, k_{ib} : Normalized normal stress intensity factors at crack tips
L : Half the distance between the rigid inclusions
 δ : Power of singularity at the edges of inclusions
 γ : Power of singularity at the crack tips
 H_1 : Hankel transform with J_1
 H_0 : Hankel transform with J_0

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