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Design and analysis of active vehicle suspension using gramian matrix based LQG control

Gramian matrix tabanlı LQG kontrolü kullanılarak aktif araç süspansiyonunun tasarımı ve analizi

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Abstract

In the present study, a quarter vehicle active suspension model with one degree of freedom designed considering only vertical forces were examined. LQG-based closed-loop suspension control was used to minimize the response of the system according to different road profiles. Sensor noises were added for the realistic simulations of the designed control system and a Kalman filter was used to filter these noises. The active suspension system was analyzed with the MATLAB simulation software package. The feedback signal and sensor location which used for the control system were determined using the system's Gramian matrix with the new approach. The LQG control system was compared to the conventional passive suspension system, according to the obtained results. In this study, three different road inputs were applied to active and passive suspension systems modeled according to several feedback signals. Although the LQG control was exposed to sensor noise, its damping ability against different road inputs was determined to be better than the passive suspension system.

Keywords: Linear systems, Optimal control theory, Kalman filtering techniques, Vehicle suspension, Quarter Car model.

Öz

Bu çalışmada sadece düşey kuvvetler dikkate alınarak tasarlanan tek serbestlik dereceli çeyrek araç aktif süspansiyon modeli incelenmiştir. Sistemin farklı yol profillerine göre tepkisini en aza indirmek için LQG tabanlı kapalı döngü süspansiyon kontrolü kullanıldı. Tasarlanan kontrol sisteminin gerçekçi simülasyonları için sensör sesleri eklenmiş ve bu gürültüleri filtrelemek için Kalman filtresi kullanılmıştır. Aktif süspansiyon sistemi MATLAB simülasyon yazılım paketi ile analiz edilmiştir. Yeni yaklaşımla kontrol sistemi için kullanılan geri besleme sinyali ve sensör konumu sistemin Gramian matrisi kullanılarak belirlendi. Elde edilen sonuçlara göre LQG kontrol sistemi geleneksel pasif süspansiyon sistemiyle karşılaştırıldı. Bu çalışmada çeşitli geri bildirim sinyallerine göre modellenen aktif ve pasif süspansiyon sistemlerine üç farklı yol girdisi uygulanmıştır. LQG kontrolü sensör gürültüsüne maruz kalmasına rağmen farklı yol girdilerine karşı sönümleme yeteneğinin pasif süspansiyon sistemine göre daha iyi olduğu belirlendi.

Anahtar kelimeler: Doğrusal sistemler, Optimal kontrol teorisi, Kalman filtreleme teknikleri, Taşıt süspansiyonu, Çeyrek Araba modeli.

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1 Introduction

In order to improve ride comfort, especially in the past ten years, some research activities are conducted on suspension [1]. The growing competition in automobile sectors has forced the industries to reinforce the posh especially the ride comfort of vehicles [2]. Suspension models are very important in other vehicles as well as cars. For example, when trains are moving, a sudden change in superstructure stiffness at transitions at the starting and ending points of tunnels or bridges causes unwanted vibrations in both the rail structure and the vehicle [3]. The most purpose of a suspension is to supply a high level of ride quality and isolate the vehicle from paved surface irregularities [4]. To understand the effect of vibration on comfort; A six-degree-of-freedom train with front and rear suspension was built [5]. It also increases the driving control of the suspension systems and keeps the wheels in contact with the ground by keeping the wheels in the correct position [6]. It is stated that the vibration to which the vehicle chassis will be exposed may cause an irritating sound, damage to the fittings in the vehicle, and health problems such as vertebral disorders and increased heart rhythm in passengers [7].

In general, suspension modeling generally consists of three main elements. These; support element, transformation spring element and damping element [8]. In a control system based on a quarter car model; Options for designing linear frequency multivariable controllers for a suspension system were presented [9]. In transportation vehicles, suspension modeling consists of three main parts. These; passive, semi-active and fully active. Passive suspensions consist of static springs and shock absorbers. The semi-active suspension consists of passive elements like spring, variable damper and doesn't add energy to the suspension [10]. The Active suspension uses separate actuators which exert an independent force on the system to enhance the ride characteristics [11]. Therefore, a variety of active control strategies has been studied by many researchers within the field of auto dynamics, for instance, adaptive control [12-13], H∞ control [14-15], backstepping control [16], linear quadratic gaussian (LQG) optimal control [17], linear quadratic regulator (LQR) control, Reliable Robust Control [18], PID control [19]. To compare control approaches; losses are quantified. LQ and LQG active suspension designs are examined in terms of stability and durability [20].

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For semi-active suspension system; for active suspension system with control unit design; A distinction was made between control unit design. The semi-active design is based on two sensors and a Kalman filter, while the active design is based on three sensors and a binary predictive Kalman filter [21]. In transportation vehicles; To determine the benefit ratio of the suspension system, a mathematical model is required. This requirement led to the determination of a mathematical model called the quarter vehicle model. This useful and simple model; It consists of wheel, spring mass, unsprung mass and sub-components [22].

This system, called the quarter vehicle model; It only detects vertical outputs on means of transportation. The simple structure of the system does not allow us to determine all outputs. But; thanks to this simple structure, the benefit rate of the system is determined accurately [23]. Data from this quadrant model completes full systems thanks to control algorithms [24]. For example, in one study, interference from the road Assuming that the effect is unknown, an adaptive controller was designed. The disturbance caused by the irregularity is modeled as the sum of different sinusoidal waves with unknown frequency, amplitude and phase values, and the observer design is made [25]. LQG is a useful approach in the field of control. Working principle; It allows the determination of a more useful controller compared to the second-order control approach. On the other hand, this approach; It also takes into account Gaussian noise in the system. In fact, many systems are exposed to different types of noise [26]. Because of this; the specified control systems must be resistant to such errors.

In this work, the LQG-based algorithm that supported optimal control theory was applied to a quarter vehicle model which has an active suspension system. The damping performance of the Quarter vehicle model was obtained with the MATLAB software package. The feedback signal and sensor location which used for the control system were determined using the system's Gramian matrix with the new approach. Finally, the performance of active and passive suspensions was compared for different road profiles (sinusoidal, stepped, and repulsive).

2 Quarter vehicle suspension model

Two-quarter, passive and active vehicle suspension models are designed in two dimensions for the study. These models are represented schematically in Fig. 1.

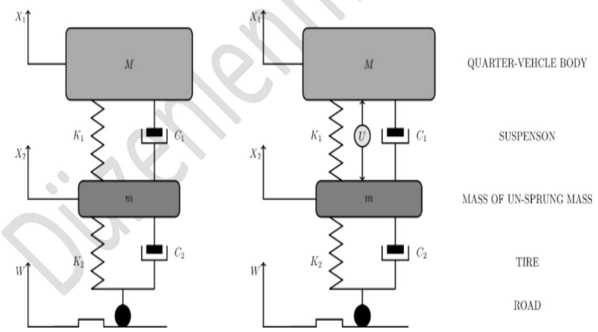

Figure 1. Passive suspension system and active suspension system.

The parameters required for the quarter vehicle model are taken from Chumjun C. (2007). In Table 1, W Road is excitation, x_1 is the vertical displacement of the sprung mass, x_2 is the vertical displacement of the unsprung mass, and U is controller output (force) which is to be controlled.

Table 1. Parameters of quarter vehicle suspension model.

Parameter description	Symbol	Value	Unite
Mass of sprung mass	M	327	kg
Mass of un-sprung mass	m	116	kg
Stiffness coefficient of the suspension	K_1	73575	N/m
Vertical stiffness of the tire	K_2	256740	N/m
Damping coefficient of the suspension	C_1	890.9	Ns/m
Damping coefficient of the tire	C_2	730	Ns/m

3 Mathematical model

Dynamic motion equations of the quarter vehicle active suspension have been obtained using Newton's laws. In the suspension models, which in motion and interaction, Newton's second and third laws have been applied for masses. In the mathematical model, only mass movements on the vertical axis and masses associated with the wheel/tire /axle shaft assembly are considered [27]. The second-order differential equations representing the dynamic motion model obtained by considering these conditions are presented as in Eqs. (1) - (2):

$$M\ddot{x} = -C_1(\dot{x}_1 - \dot{x}_2) - K_1(x_1 - x_2) + U \tag{1}$$

$$m\,\ddot{x}_2 = C_1(\dot{x}_1 - \dot{x}_2) + K_1(x_1 - x_2) + C_2(\dot{W} - \dot{x}_2) + K_2(W - x_2) - U \tag{2}$$

To transform Eq. (1) and Eq. (2) into state-space form, the state variables are arranged as follows. Here x_1-x_2 is the suspension travel and spring-mass velocity.

$$z_1 = x_1 \tag{3}$$

$$z_2 = \dot{x}_1 \tag{4}$$

$$z_3 = x_1 - x_2 \tag{5}$$

$$z_4 = \dot{x}_1 - \dot{x}_2 \tag{6}$$

The general form of the state-space equations given below have been used for the transformation of Eq.(1) and Eq.(2) into state-space form.

$$\dot{z} = Az + Bu \tag{7}$$

$$y = Cz + Du \tag{8}$$

Derived from Eqs.(7-8) and Eqs.(1-6), state matrix A, input matrix B, output matrix C and the direct transmission matrix D have been given below.

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$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(C_1.C_2)}{(M.m)} & 0 & \frac{C_1}{M}.\left(\frac{C_1}{M}+\frac{C_1}{m}+\frac{C_2}{m}\right)-\frac{K_1}{M} & -\left(\frac{C_1}{M}\right) \\ \frac{C_2}{m} & 0 & -\left(\frac{C_1}{M}+\frac{C_1}{m}+\frac{C_2}{m}\right) & 1 \\ \frac{K_2}{m} & 0 & -\left(\frac{K_1}{M}+\frac{K_1}{m}+\frac{K_2}{m}\right) & 0 \end{bmatrix} \quad (9)$$

$$B=\begin{bmatrix} 0 & 0 \\ \frac{1}{M} & \frac{C_1.C_2}{M.m} \\ 0 & -\frac{C_2}{m} \\ \frac{1}{M} & -\frac{K_2}{m} \end{bmatrix} \Rightarrow \text{B}=[B_1 \quad B_2] \quad (10)$$

$$C=[0 \quad 0 \quad 1 \quad 0] \quad (11)$$

$$D=[0 \quad 0] \quad (12)$$

4 Control ve methodology

To form the LQG regulator, the Kalman filter simply has been connected to the LQ-optimal gain K and the integrator for the following reference value (Fig 2). The system has been operated with input (w), which represents the road profile in the control diagram given in Fig.2. The noise signal (n) is sensor sounds. Kalman filter is used to filter the noise signals and LQR control is used to control the signal coming out of the filter.

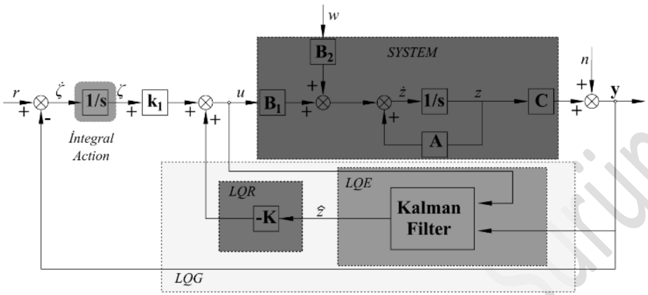

Figure 2. LQG controller with integral action and reference input.

Considering the parameters w, n, and LQR control, the state-space equations expressed in (Eqs (7)-(8)) take the form below.

$$\dot{z}=Az+B_1u+B_2\text{w} \quad (13)$$

$$y=Cz+I_nu \quad (14)$$

In these equation in is nxn grade unit matrix. The input sign u and the error function ζ have been expressed in the following equations.

$$u=\zeta k-\hat{z}K \quad (15)$$

$$\dot{\zeta}=I_r-y \quad (16)$$

Eqs.(15) and Eqs.(14) are arranged by placing them in Eq.(13) and Eq.(16) respectively, and the following equations have been obtained.

$$\dot{z}=Az+B_1(\zeta k-\hat{z}K)+B_2w \quad (17)$$

$$\dot{\zeta}=I_r-Cz-I_nn \quad (18)$$

For Eq.(7) and Eq.(8), in Eq.(18), since r (t) - r (+ ∞) = 0 for t → +∞, the state-space matrices can be written in the form below.

$$\begin{bmatrix}\dot{z}\\ \dot{\zeta}\end{bmatrix}=\begin{bmatrix}A-B_1K & B_1k\\ -C & 0\end{bmatrix}\begin{bmatrix}z\\ \zeta\end{bmatrix}+\begin{bmatrix}B_2 & 0\\ 0 & I_n\end{bmatrix}\begin{bmatrix}w\\ n\end{bmatrix} \quad (19)$$

$$y=[C \quad 0]\begin{bmatrix}z\\ \zeta\end{bmatrix}+[0 \quad I_n]\begin{bmatrix}w\\ n\end{bmatrix} \quad (20)$$

4.1 Optimum feedback gain

The LQG regulator consists of an effective gain in-state feedback and a Kalman state estimator, and it is necessary to independently design these two elements. In the full state feedback control system, In case (A, B) parameters are checked, it is possible to determine the equations of the closed-loop system (A-BK) using the equation u = -K z with the pole addition method. Given a controllable system; for an observable system with either full state measurements or full state estimation, there are many options for stabilizing the system with the pole placement method [26]. Choosing data too effectively may cause the system to respond too effectively to noise and distortions. In order to prevent such undesirable situations, gain values must be determined at optimum levels. In this way, the system is optimized against overreactions. The following cost function (Eq.21) is used to balance these criteria.

$$J(t)=\int_0^t z(\tau)^T Q\, z(\tau)+\, u(\tau)^T R\, u(\tau)\, d\tau \quad (21)$$

Given in the above equation; The Q matrix represents the deviation of the state variables from zero and the R matrix represents the weight of the system inputs. Q is defined as half positive and R is defined as positive. These diagonal matrices can be adjusted according to the situation conditions. The first term on the right side of the equation solves the error between the initial and final state and second-semester expenditure controls the energy of the signal [27]. By convention, choose the weight matrices of the LQR controller based on the trial and error methods to determine the best state feedback controller gain [28].

4.2 The Kalman filter

Considering the pole placement method, LQ-Optimal state feedback u=-kz cannot be applied without full state feedback measurement [29]. It is more possible to predict limited noise conditions than to determine all of the state variables. This probability depends on the value measured by the observability gramian (A;C), which can only determine mathematical probability [26]. The Kalman filter is more preferred because it can detect these external effects with less deviation. Control system with Kalman filter. It tries to estimate the actual output (y) and state of the system by looking at the input (u) given to the system and the measured noisy and output (y) (Fig.3).

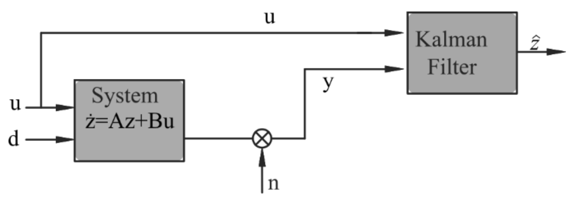

Figure 3. Kalman filter working diagram.

Q and R parameters, called noise covariance matrices, determine the filter's performance. Kalman filter gives very good results if these values are used in cases where noise covariances are known exactly. However, in a real problem, it is generally not possible to determine and calculate these values.

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In practice, these values can be chosen by concluding that if it gets Q>R the problems related to the mathematical model are more dominant. Otherwise, problems related to measurements are more dominant. Kalman filter uses a cost function to minimize the amount of error between the state variables and the filtered state variable from the system (Eq. 22).

$$\lim_{t\to\infty} E((z-\hat{z})\,(z-\hat{z})^T) \qquad (22)$$

The equation given above determines the effects required to achieve the optimum balance by reducing noise. So, the Kalman filter is called linear quadratic estimator (LQE). Additionally, it has an integrated formulation with LQR.

4.3 Gramians and degrees of controllability / observality

Expected in the controllability and observability tests of the control system; The degree of controllability C and observability matrix O is equal to the system grade (Eqs. (23-24)).

$$C = [B \quad AB \quad A^2B \quad A^{n-1}B] \qquad (23)$$

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^{n-1} \end{bmatrix} \qquad (24)$$

Since some x state variables can be more easily controlled or predicted than others, there are degrees of controllability and observability. To determine which state variable is more or less controllable, an analysis of controllability Gramian's eigenvalues and eigenvectors is required. (Eqs (25-27)) [28].

$$W_c(t) = \int_0^t e^{A\tau} B\, B^T e^{A^T\tau} d\tau \qquad (25)$$

Similarly, observability Gramian is expressed as follows;

$$W_o(t) = \int_0^t e^{A^T\tau} C^T C\, e^{A\tau} d\tau \qquad (26)$$

The eigenvalue equality of both Gramians is;

$$W_c\xi = \lambda_c\xi \quad \text{and} \quad W_o\xi = \lambda_o\xi \qquad (27)$$

expressed in the form.

The obtained values and vectors are sorted so that the vectors corresponding to larger values can be checked. Grammars can be visualized as an ellipsoid in state space, with major axes given by directions hierarchically ordered for controllability or observability (Fig. 4).

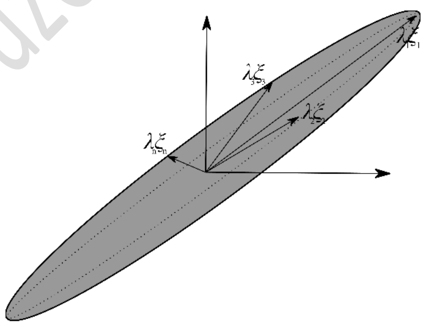

Figure 4. Gramian ellipsoid.

Since the eigenvalues of these hierarchically sortable state variables depend on the input and output matrices B and C, they are often used to evaluate the observability/controllability of the placement of a particular sensor and actuator [29].

5 Road profile

Three different road profile scenarios are considered to evaluate the performance of the designed controller.

5.1 The unit step function

In this scenario; road excitation a function of time is modeled as a cosine bump road profile. The road entry profile is given as the cosine function given by the equation below.

$$w = \begin{cases} -\frac{H}{2}\left(\cos\left(2\pi\frac{x}{L}\right) - 1\right) \dots if\ 0 < x < L \\ 0 \dots else \end{cases} \qquad (28)$$

H and L are the height and width of the cosine bump. v is the speed at which the car goes through bumps; x=v.t, t is the time it takes the car to pass the speed bump [30].

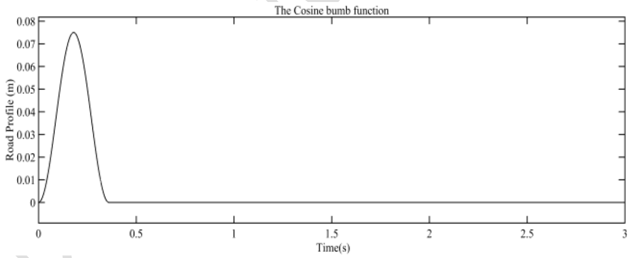

Figure 5. Cosine bump road profile.

5.2 The unit step function

In this scenario; the stimulation signal is given to the system as a unit step function.

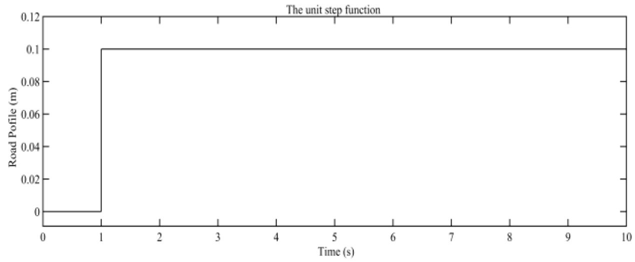

Figure 6. Unit step function road profile.

5.3 The sinusoidal wave functions

The suspension system is finally stimulated with a sinusoidal wave whose function is given below [31].

$$w = 0.006\ Sin(2\pi t) \qquad (29)$$

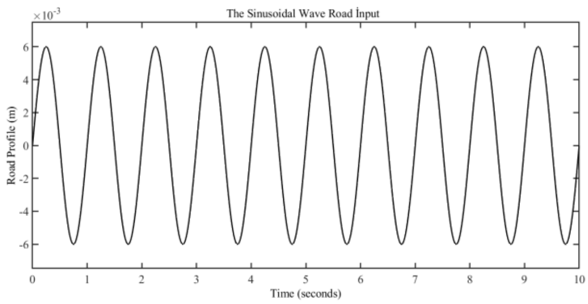

Figure 7. Sinusoidal wave function road profile.

6 Performance declaration

When a suspension system is designed, comfort, suspension movement, dynamic tire strength, and handling as performance parameters considering. Driving comfort RMS is characterized by spring-mass acceleration [4]. According to ISO 2631-1 [32]. If weighted RMS appears Passengers feel that the mass acceleration is less than 0.315 m/s² very comfortable. At least 0.127 m of suspension travel is required, and the maximum spring-mass acceleration should not be increased by 4.5 m/s² to avoid hitting the suspension stop 0.058m. Also, applies to roads with unfixed quality the displacement is 0.07m. [32].

7 Simulation and experimental results

Eq.(19) and Eq.(20) obtained by using the equations of motion of the quarter car active suspension system were solved as a script file in MATLAB as the pole placement problem. In addition, a Simulink block diagram control model was created with MATLAB /SIMULINK to see the effects of the obtained results on the system. To confirm the efficiency and accuracy of the design for the enhanced LQG controller, a sample simulation was made to evaluate the proposed controller for the active suspension system. Comparative analysis of this simulation using an LQG controller was performed with a passive suspension system. First, controllability Gramian's eigenvalues and eigenvectors were obtained to determine which state variable was more controllable to control the linear actuator. Thus, the controllability of the suspension system Gramian matrix was obtained as below.

$$W_c = 1.0e{+}04 \begin{bmatrix} 0.0065 & 0.0000 & 0.0056 & -0.0326 \\ 0 & 0.1806 & -0.0012 & 0.2321 \\ 0.0056 & -0.0012 & 0.0086 & 0.0385 \\ -0.0326 & 0.2321 & 0.0385 & 3.0713 \end{bmatrix} \quad (30)$$

The Eigenvalue values of the controllability Gramian matrix obtained are presented below.

$$\mathrm{eig}(W_c) = 1.0e{+}04 \begin{bmatrix} 0.0009 \\ 0.0132 \\ 0.1622 \\ 3.0906 \end{bmatrix} \quad (31)$$

Hierarchically controllability order according to this matrix is Z_4, Z_3, Z_2, Z_1, and it shows which system variable is easier to control with one unit of energy given to the system to control the system. By looking at the Eigenvalue values of the matrix above, the feedback was made in the Simulink environment according to the Z_4 state variable, which is the speed of the distance between the wheel and the chassis. Therefore, the measuring sensor must be placed according to the state variable Z_4. To compare the Z_4 state variable with the system performance, the system outputs of the state variable Z_3 will also be evaluated.

The schematic representation of the active suspension system (Fig.8), which is rearranged according to these situation variables, is given below.

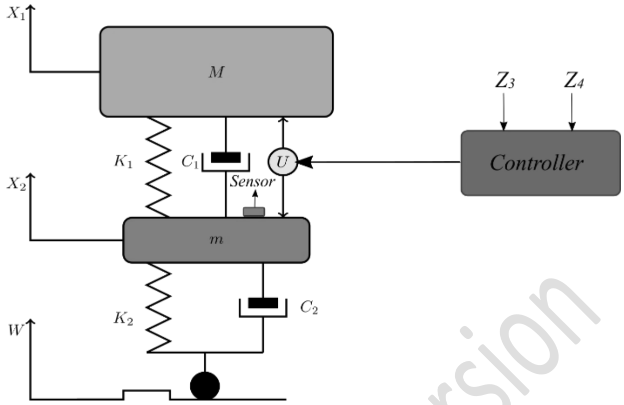

Figure 8. Sensor placement for suspension.

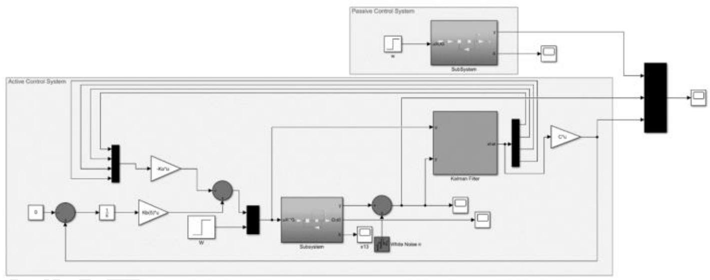

Figure 9. MATLAB /Simulink control model block diagram.

In theory, for most systems, with a suitable gain matrix, the system poles can be placed anywhere. A fast and stable device response can be obtained using this technique. However, actual performance is limited by physical hardware. The best effect can be achieved by optimizing the response speed and control intensity. In order to achieve this goal, the linear quadratic regulator (LQR) method is used, and the LQR is obtained through the cost function given by Equation (21) [33]. Weight matrices of the cost function are scattered as follows to achieve the desired performance.

$$\begin{matrix} 10^6 \le Q_{11} \le 10^7 \ , \ 10^6 \le Q_{22} \le 10^7 \\ 10^8 \le Q_{44} \le 10^9 \ , \ 10^8 \le Q_{33} \le 10^9 \end{matrix} \quad 1 \le R \le 2 \quad (32)$$

The state feedback controller design aimed to find the K gain matrix. The gain matrix and the integral gain value that was obtained using the continuous-time control system model under real constraints according to the determined cost function were obtained as below (Eq.33, Eq.34).

According to Z_3;

$$\begin{matrix} K = 1.0e + 05\ [1.8258 \quad 0.0138 \quad \text{-}1.3545 \quad 3.3295] \\ k = 1.0e{+}06\ [\text{-}1.0541] \end{matrix} \quad (33)$$

According to Z_4;

$$\begin{matrix} K = 1.0e + 05\ [1.8320 \quad 0.0028 \quad \text{-}2.4227 \quad 3.3348] \\ k = 1.0e{+}06\ [\text{-}1.0541] \end{matrix} \quad (34)$$

A random signal between 1.247×10^{-2} and -1.140×10^{-2} amplitude is applied to the output of the system state-space model to represent the noise in the sensor. (Fig.10).

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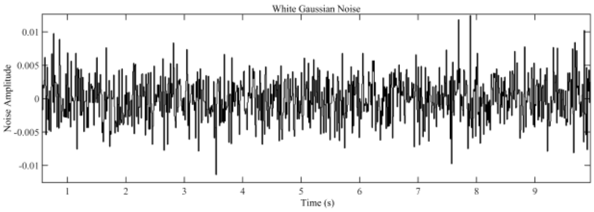


Figure 10. Block diagram created for road functions.

MATLAB Kalman filtering and Kalman function block were used to filter the noisy signal from the system state-space and to obtain clean output signals. When creating the Kalman filter, it was accepted that the noise in the signal coming from the sensors was more dominant and the Q_N and R_N parameters were chosen according to the acceptance. In this case, noise covariances were determined as below.

$$Q_N = 0.1\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \text{ and } R_N = 100 \tag{35}$$

The control system created in Simulink was driven by three different road functions previously described. For the first road function, H=0.075 m, L=0.5 m, v= speed of the vehicle passing over the bump was taken as 5 km/h [29]. A separate block diagram was created to drive the control system of the road functions (Fig.11).

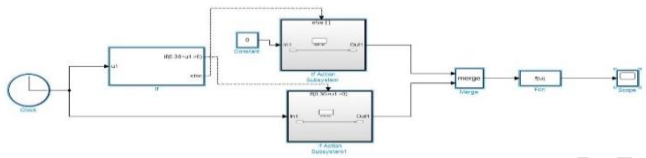
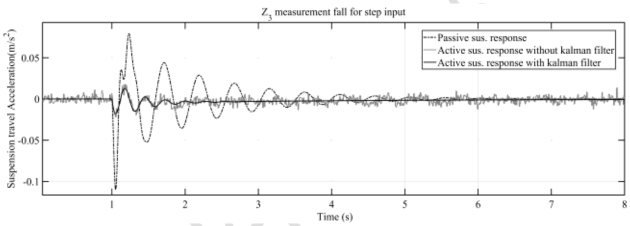
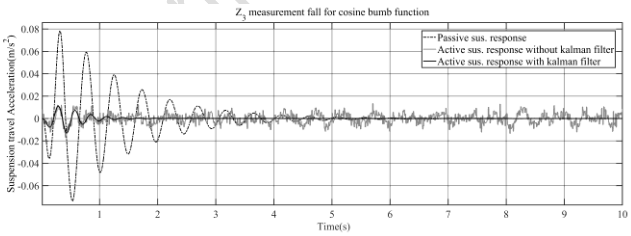


Figure 11. Road functions block diagram.

Active and passive suspension systems were driven by three different road functions and two different feedbacks were provided. The simulation results of the systems were given below (Figs.12-13).



(a)



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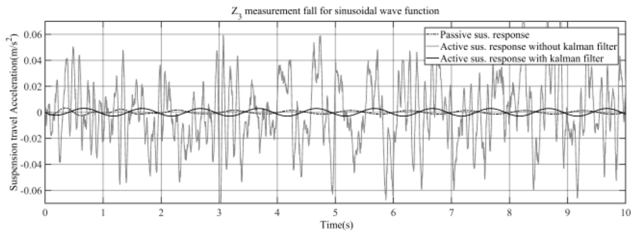
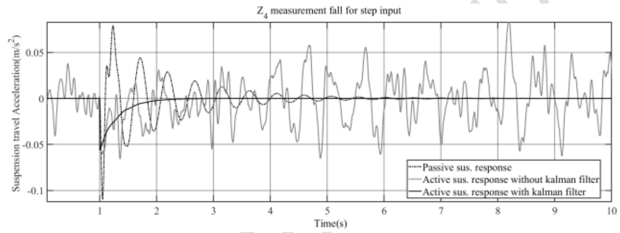
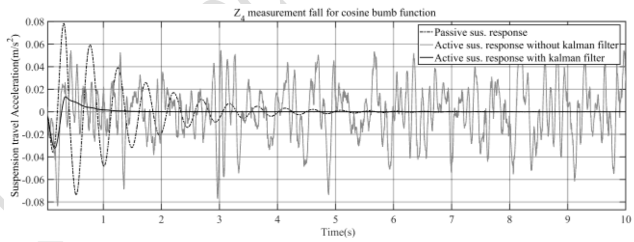


(c)

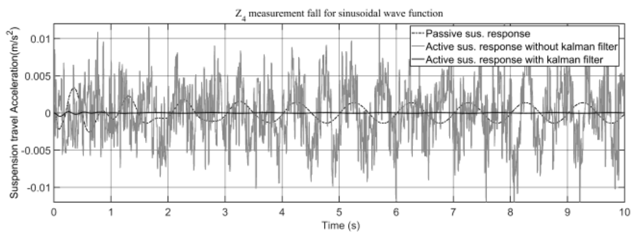
Figure 12. Input parameters for Z_3 measurements (a: time response of step, b: cosine, c: sinusoidal).



(e)



(f)



(g)

Figure 13. Input parameters for Z_4 measurements (e: time response of step, f: cosine, g: sinusoidal).

Figs.12-13 show that the ability of the active suspension system using the LQG controller to absorb different road inputs is more effective than the traditional passive suspension system. In addition, the performance change of the active suspension system according to the different sensor outputs selected based on the eigenvector of the Wc gramien matrix is also shown. The figures clearly show that the Kalman filter optimizes the sensor noises and thanks to this filter, the control system is not affected by these noises.

8 Conclusion

The main objectives of this article are to demonstrate the superiority of the active control system, which uses the LQG controller exposed to sensor noises over the passive suspension system. In the study, the controllability is expressed using the eigenvectors of the gramien matrix how the LQG controller performance changes as the new approach.

Also, passive suspension systems without any controller and active suspension systems with LQG controllers were both modeled and simulated using Matlab / Simulink environment to increase driving comfort. The simulation results showed that the designed active system oscillates less than the passive system and could be improve driving quality by minimizing acceleration.

In addition, the degree of controllability of system variables is demonstrated using the eigenvectors of the controllability matrix and when the measurement signal is determined according to this degree, the performance of the control system is shown with graphics. Similarly, the LQG controller is tested for three different road conditions under the sensor noise using MATLAB, and its flexibility and superiority compared to the passive system are shown.

9 Author contribution statement

In the conducted study, author 1; He contributed to the formation of the idea, literature review, creation of the mathematical model and evaluation of the findings. Author 2; contributed to the planning and analysis processes of the research. Author 3; contributed to data analysis and design of presented figures.

10 Ethics committee approval and conflict of interest statement

“There is no need to obtain ethics committee permission for the article prepared.”

“There is no conflict of interest with any person/institution in the prepared article.”

11 References

[1] Bashir AO, Rui X, Abbas LK, Zhou Q. “MR-damped Vehicle Suspension Ride Comfort Enhancement Based on Advanced Proportional-Integral-Differential Sliding Mode Control”. *Control Engineering and Applied İnformatics*, 20(4), 11-21, 2018.

[2] Rajeswari K, Lavanya S, Lakshmi P. “Grey Fuzzy sliding Mode controller for Vehicle suspension System”. *Control Engineering and Applied İnformatics,* 17(3), 12-19, 2015.

[3] Ulu A, Metin M. “Control of railway vehicle vibrations due to the effect of different superstructure stiffness in transition zones with rail irregularities”. *Pamukkale University Journal of Engineering Sciences,* 26(4), 709-719, 2020.

[4] Yıldız AS, Sivrioglu S. “Constrained adaptive backstepping control of a semi-active suspension considering suspension travel limits”. *Asian Journal of Control*. 23(3), 1-14, 2020.

[5] Khadanga KK, Lee HH. “Robust stability of active suspension lqg control for railway vehicle with parametric fluctuations”. *2018 International Conference on Information and Communication Technology Robotics (ICT-ROBOT)*, Japan, Kitakyushu, 06 – 08 September 2018.

[6] Nagarkar M, Patil GJV. “Multi-Objective Optimization of LQR Control Quarter Car Suspension System using Genetic Algorithm”. *FME Transactions*, 44(2), 187-196, 2016.

[7] Rezazadeh A, Moradi H. “Design of optimum vibration absorbers for a bus vehicle to suppress unwanted vibrations against harmonic and random road excitations”. *Scientia Iranica*, 28(1), 241-254, 2021.

[8] Goodarzi A, Khajepour A. *Vehicle suspension system technology and design*. Waterloo, Canada, Morgan & Claypool Publishers, 2017.

[9] Mohamed ME, Zuhair A. “Linear Quadratic Gaussian Control of a Quarter-Car Suspension”. *Vehicle System Dynamics.* 32(6), 479-497, 1999.

[10] Bharali J, Buragohain M. “A comparative analysis of PID, LQR and Fuzzy logic controller for active suspension system using 3 Degree of Freedom quarter car model”. *IEEE 1 St International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES),* Delhi, India, 04 – 06 July 2016.

[11] Alexandru C, Alexandru P. “Control Strategy for an Active Suspension System”. *World Academy of Science, Engineering and Technology*, 5(7), 1204-1209, 2011.

[12] Huang Y, Na J, Wu X, Gao G. “Approximation-free control for vehicle active suspensions with hydraulic actuator”. *IEEE Transactions on Industrial Electronics*, 65(9), 7258–7267, 2018.

[13] Pan H, Sun W, Gao H, Jing X. “Disturbance observer-based adaptive tracking control with actuator saturation and its application”. *IEEE Transactions on Automation Science and Engineering*, 13(2), 868–875, 2015.

[14] Sammier D, Sename O, Dugard, L. “Skyhook and H8 Control of Semi-active Suspensions: Some Practical Aspects”. *Vehicle System Dynamics*, 39(4), 279-308, 2003.

[15] Du H, Sze KY, Lam J. “Semi-active H-infinity control of vehicle suspension with magneto-rheological dampers”. *Journal od and Vibration*. 283(3). 981-996, 2005.

[16] Sun W, Gao H, Kaynak O. “Adaptive backstepping control for active suspension systems with hard constraints”. *IEEE/ASME Transactions on Mechatronics*, 18(3), 1072–1079, 2012.

[17] Gomonwattanapanich O, Pannucharoenwong N, Rattanadecho P, Echaroj S, Hemathulin S. “Vibration Control of Vehicle by Active Suspension with LQG Algorithm”. *International Journal of Automotive and Mechanical Engineering*, 17(2), 8011-8018, 2020.

[18] Gudarzi M. “Reliable robust controller for half-car active suspension systems based on human-body dynamics”. *Facta Universitatis Series Mechanical Engineering,* 14(2), 121-134, 2016

[19] Vu T, Dung D, Trang N, Hai P. “Analytical design of PID controller for enhancing ride comfort of active vehiclesuspension system”. *In: International Conference on System Science and Engineering (ICSSE),* Ho Chi Minh City, Vietnam, 21 – 23 July 2017.

[20] Ulusoy AG, Hrovat D, Tseng T. “Stability robustness of LQ and LQG active suspensions”. *Journal of Dynamic Systems, Measurement, and Control.* 116(1), 123-131, 1994.

[21] Muhammed A, Gavrilov A. “Managing the handling–comfort contradiction of a quarter-car system using Kalman filter”. *Transactions of the Institute of Measurement and Control*. 43(10), 2292-2306, 2021.

[22] Chetan R, Both-Rusu R, Dulf E, Festila C. “Physical model of a quarter-car active suspension system”.*18th International Carpathian Control Conference (ICCC),* Sinaia, Romania, 28 – 31 May 2017.

[23] Krauze P, Kasprzyk J. “Neural network based LQ control of a semiactive quarter-car model”. *18th International Conference on Methods & Models in Automation & Robotics (MMAR),* Międzyzdroje, Poland, 26 – 29 August 2013.

[24] Zohoor H. “Optimal Preview Control Design of an Active Suspension Based on a Full Car Model”. *Scientia Iranica*, 10(1), 23 - 36, 2003.

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[25] Karasız G, Baştürk Hİ. “Design of an controller under unknow disturbance input for active suspension systems. *Pamukkale University Journal of Engineering Sciences*, 24(8), 1403-1408, 2018.
[26] Labane C, Zemalache MK. “Aircraft control system using LQG and LQR controller with optimal estimation-kalman filter design. *3th International Symposium on Aircraft Airworthiness*, Nairobi, Kenya, 7 – 11 April 2014.
[27] Afshar K, Javadi A, Jahed-Motlagh MR. “Robust H∞ control of an active suspension system with actuator time delay by predictor feedback”. *IET Control Theory & Applications*, 12(7), 1012-1023, 2018.
[28] Brunton SL, Kutz JN. “Machine Learning, Dynamical Systems, and Control, Linear Control Theory”. Data-Driven Science and Engineering, 1 - 76, Cambridge, England, Cambridge University, 2019.
[29] Manohar K, Kutz JN, Brunton SL. “Optimal sensor and actuator placement using balanced model reduction”. IEEE Transactions on Automatic Control, 67(4), 2108 - 2115 2018.
[30] Kanjanavapastit A, Thitinaruemit A. "Estimation of a speed hump profile using quarter car model". *Procedia Social and Behavioral Sciences*, 88, 265-273, 2013.
[31] AN, NAIS; SI, S. Mechanical Vibration and Shock– Evaluation of Human Exposure to Whole Body Vibration– Part 4: Guidelines for the Evaluation of the Effects of Vibration and Rotational Motion on Passenger and Crew Comfort in Fixed-Guideway Transport Systems. 2007.
[32] Baumal AE, McPhee JJ, Calamai PH. “Application of genetic algorithms to the design optimization of an active vehicle suspension system”. *Computer Methods in Applied Mechanics and Engineering*, 163, 87-94, 1998.
[33] Kizir S. “Real time full state feedback control of a Seesaw system based on Lqr”. *Journal Of Polytechnıc*, 22(4), 1023-1030, 2019.

Düzenlenmemis Sürüm - Uncorrected Version