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# Avoid maximum cost method for determining the initial basic feasible solution of the transportation problem

## Ulaştırma probleminin başlangıç uygun çözümünün belirlenmesi için en büyük maliyetten kaçınma yöntemi

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**Abstract**

*The transportation problem is an optimization problem related to determining the transportation plan that will ensure the transportation of products from supply points to demand points with minimum total cost. Although this problem can be modeled as a linear programming model because of its special structure, it is usually solved in two phases: finding the initial basic solution and finding the optimal solution. Thus, finding a good initial solution is important, especially in large problems since it will reduce the number of steps required in the second phase. To date, many approaches have been developed to find the initial basic solution. In this study, a new method called avoid maximum cost method is proposed for determining the initial basic solution of the transportation problem. The advantage of this algorithm is that it is easy to understand and implement. The avoid maximum cost method is applied to test problems and compared with six well-known initial solution methods. The results show that the proposed method produces a consistent and very good initial basic feasible solution. In addition, because of its simplicity, this method can be used as an alternative method for an initial basic feasible solution besides well-known methods in teaching.*

**Keywords:** Transportation, Initial solution, Approximation method.

**Öz**

*Ulaştırma Problemi, ürünlerin arz noktalarından talep noktalarına minimum toplam maliyetle taşınmasını sağlayacak taşıma planının belirlenmesi ile ilgili bir optimizasyon problemidir. Bu problem, özel yapısı nedeniyle bir doğrusal programlama modeli olarak modellenebilse de genellikle başlangıç temel çözümünü bulma ve en uygun çözümü bulma olmak üzere iki aşamada çözülür. Bu nedenle, özellikle büyük problemlerde, ikinci aşamada gereken adım sayısını azaltacağından, iyi bir başlangıç çözümü bulmak önemlidir. Bugüne kadar başlangıç temel çözümünü bulmak için birçok yaklaşım geliştirilmiştir. Bu çalışmada, ulaştırma probleminin başlangıç çözümünün belirlenmesi için maksimum maliyetten kaçınma yöntemi adı verilen yeni bir yöntem önerilmiştir. Bu algoritmanın avantajı, anlaşılması ve uygulanmasının kolay olmasıdır. Maksimum maliyetten kaçınma yöntemi test problemlerine uygulanmış ve iyi bilen altı başlangıç çözüm yöntemi ile karşılaştırılmıştır. Sonuçlar önerilen yöntemin tutarlı ve iyi başlangıç uygun çözümler ürettiğini göstermektedir. Ayrıca, çok basit olması nedeniyle bu yöntem öğretimde çok bilinen yöntemlerle birlikte başlangıç uygun çözümlerin bulunmasında alternatif olarak kullanılabilir.*

**Anahtar kelimeler:** Ulaştırma, Başlangıç çözüm, Yaklaşım yöntemi.

## 1 Introduction

As the local and global competition increase, logistics has become a very important part of production. All companies, whether operating in the production or service sector must transport some materials or products to their customers [1]. In addition to increasing competition, the developments in information and communication technologies have led to the delivery of final products, in-process inventory, raw materials, or related information from the supply points to the demand points in an efficient and low-cost way [2].

The transportation problem (TP) is a special case of the minimum cost network flow problem where all nodes are either supply or demand nodes. The goal of the problem is to find the shipments from supply points to demand points to minimize the total transportation costs. This problem appears very often in practice in a variety of contexts [2],[3].

TP can be modeled as linear programming (LP) and solved efficiently by using the simplex algorithm. The simplex algorithm is performed in two phases. In the first phase, an initial basic feasible solution (IBFS) is found, and in the second phase, this initial solution is improved by changing the basic variables until the optimal solution is found. Therefore, finding a good IBFS is important, especially in large problems, since it will decrease the number of iterations required in the second phase [4]. There are several methods available in the literature to find the IBFS for TP [5]. North-West corner (NWC), least cost method (LCM), row-minima (RM), column-minima (CLM), and Vogel's approximation method (VAM) are among the most popular methods. Although the performance of these methods may depend on the problem, some methods, such as NWC, require the least computation but usually yield worse initial solutions and some methods, such as VAM require more computation but may find an initial solution close to the optimal solution. Therefore, a new method that both produces

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a better initial solution and requires less computation is necessary.

In this study, a new solution method called Avoid Maximum Cost Method (AMCM) is proposed to find an IBFS for TP. The logic of the proposed method is very simple. To avoid making an assignment to the cell that has the maximum cost, an assignment is made to the minimum cost cell in a row or column where the maximum cost cell is located. This process is repeated until all the assignments are completed. This is a very simple method since it does not require any calculations. Therefore, it can be used as an alternative method in the lectures besides the well-known methods such as NWC, LCM, and VAM.

The rest of the paper is organized as follows. In Section 2, we give an overview of existing literature. In Section 3, we briefly discuss TP problems and give the mathematical model of the classical TP. In Section 4, we describe our proposed method in detail. In Section 5 we compare the performance of the AMCM with the well-known methods used for finding IBFS of TP. Finally, the last section concludes the paper.

## 2 Literature review

There are many methods developed in the literature to find better IBFS. Kirca and Satir [6] suggested a heuristic method called the Total Opportunity-Cost Method (TOM) for determining an IBFS for the transportation problem. Mathirajan and Meenakshi [7] offered a solution method that combines the total opportunity cost (TOC) with two variants of VAM. They have shown that the basic version of VAM combined with the total opportunity cost (VAM-TOC) provides very efficient initial solutions. Korukoglu and Ballı [8] proposed an improved version of VAM called IVAM, which uses the total opportunity cost and considers alternative allocation costs. Khan [9] considered the highest three-pointer costs calculated by taking the highest and next highest cost difference for each row and each column while determining the cell to be assigned. The allocation is made to the cell with the lowest transportation cost possible. Other studies [10]-[13] based on total opportunity cost are also included in the literature. Mhlanga et al. [14] proposed a novel approach that combines the North-West corner approach with knowledgeable and innovative cost matrix manipulation to generate efficient solutions. Das et al. [15] developed a method called Logical Development of Vogel's Approximation Method (LD-VAM) to deal with the situation where two or more rows or columns have the highest penalty cost when solving the problem with VAM. Can and Koçak [16] proposed a method called Tuncay Can Approach Method (TCM). In this method, the geometric average of the transportation costs in the transport table is taken and the cell with the closest to this average cost is assigned by considering demand and production constraints. Ahmed et al. [17] suggested an iterative approach called Allocation Table Method (ATM) based on the allocation table. In this method, an assignment is made to the cell that has the lowest demand or supply. Karagul and Sahin [2] proposed a novel method that considers both supply and demand coverage ratios with transportation costs. Khan et al. [21] developed a method called TOCM-SUM to find IBFS of the transportation problem. In this method, they first construct the total opportunity cost matrix by subtracting minimum values from each row and column and calculate the pointer cost by taking the sum of all the entries in the related row or column. Then, the maximum amount is assigned to the lowest cost cell corresponding to the highest indicator cost. Babu et al. [22] proposed an Improved Vogel Approximation Method (IVAM) to overcome some of the limitations and computational blunders of VAM. Hossain et al. [23] developed a new algorithm named TOCM-MEDM to find the initial basic feasible solution of a balanced transportation problem. Jamali et al. [24] have developed a method called "the minimum demand method (MDM)". In the method, assignments are made to the minimum value in the demand line, in the case of equality, the least costly demand in the relevant column is selected. Amaliah et al. [25] proposed a new method called "Total Opportunity Cost Matrix-Supreme Cell (TOCM-SC)" to find IBFS for the transportation problem. In the TOCM-SC method, which starts with the TOCM matrix, assignments are made by considering the Row Supreme (RS) and Column Supreme (CS) values. Sam'an and Ifriza [26] developed a new solution approach combining the Total Difference Method (TDM) and Karagul-Sahin Approximation Method (KSAM) algorithm. In addition to these studies, studies [27]-[29] for the solution of fuzzy transportation problems are also included in the literature.

## 3 The transportation problem

The classical transportation problem was first formulated by Hitchock in 1941 [18] and different solution methods have been developed since then. The problem can be described as follows. There are $m$ suppliers that produce a single product and $n$ customers are demanding this product. Each supplier $i$ can supply $a_i$ units of product, and each customer $j$ demands $b_j$ units of product and the unit transportation cost from supplier $i$ to customer $j$ is $c_{ij}$. This problem can be represented as a network in Figure 1 [19].

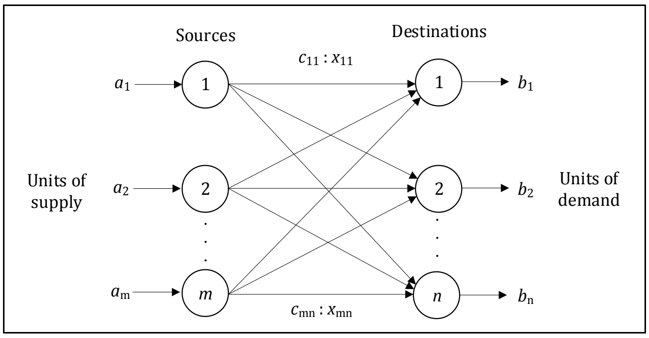


Figure 1. Network representation of the transportation model.

The mathematical model of the classical transportation problem is given as follows [20].

$$Min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij} \tag{1}$$

s.t.

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2,3,\dots,m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1,2,3,\dots,n \tag{3}$$

$$x_{ij} \geq 0 \qquad i = 1,2,3,\dots,m \;\; j = 1,2,3,\dots,n$$

Where $x_{ij}$ is the amount of product shipped from supplier $i$ to customer. The objective function (1) minimizes the total transportation cost. Constraints (2) are called capacity constraints and ensure that the capacities of suppliers are not exceeded. Constraints (3) are called demand constraints and guarantee that all the demands are satisfied. [20]. If the total demand is equal to the total capacity of the suppliers, this problem is called the balanced transportation problem.

## 4 The avoid maximum cost method

In this study, we offer a new method called Avoid Maximum Cost Method (AMCM) that finds an IBFS for a classical transportation problem. The method is based on making the assignment to either a row or a column in a way that assignment to the cell that has the highest costs will be avoided in the further steps. The steps of the proposed methods are described as follows. All the steps are performed on transportation tables.

***Step 1:*** Find the cell that has a maximum unit transportation cost and select the row and the column corresponding to this cell. If there is more than one alternative, select all the rows and columns corresponding to these alternatives,

***Step 2:*** Find the cell with the lowest unit transportation cost within the rows and columns selected in Step 1. If there is more than one candidate, select the cell whose row or column contains the highest cost,

***Step 3:*** Make the maximum assignment to the cell considering the remaining row and column capacity. After the assignment, remove the row or column whose remaining capacity is zero, if both the row and column capacity is zero, then remove the column only,

***Step 4:*** If there is only one row or column left, go to Step 5, otherwise go to Step 1,

***Step 5:*** Make the necessary assignments for the last row or column.

Note that the transportation problem must be balanced to apply this method.

## 5 Illustrative example

To explain the AMCM method, we consider the transportation problem that has four suppliers ($S_1$, $S_2$, $S_3$, $S_4$,) and four demands points ($D_1$, $D_2$, $D_3$, $D_4$,). All the parameters of the problem are given in Table 1.

Table 1. A numerical example.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|---|
| $S_1$ | 10 | 3 | 8 | 3 | 185 |
| $S_2$ | 6 | 4 | 6 | 3 | 175 |
| $S_3$ | 1 | 1 | 1 | 4 | 140 |
| $S_4$ | 10 | 6 | 7 | 2 | 30 |
| Demand | 110 | 200 | 100 | 120 | |

Since this problem is balanced, we can construct the transportation table given as in Table 2.

***Step 1:*** The maximum unit cost is 10 in cells ($S_1$, $D_1$) and ($S_4$, $D_1$). Thus, we select row S1 and column $D_1$, and row $S_4$, corresponding to these cells. These rows and columns show where the first possible assignment will be made,

***Step 2:*** The minimum unit cost in these selected rows and columns is 1 in cell ($S_3$, $D_1$) at column $D_1$(see Table 2),

***Step 3:*** The maximum assignment for cell ($S_3$, $D_1$) is 110 units. After this assignment, the demand of $D_1$ is fully satisfied we remove column $D_1$ from the table,

***Step 4:*** Since there are more than one row and column, we go to Step 1.

Table 2. Transportation table for the first iteration.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|---|
| $S_1$ | **10** | 3 | 8 | 3 | 185 |
| $S_2$ | 6 | 4 | 6 | 3 | 175 |
| $S_3$ | **1** [110] | 1 | 1 | 4 | 140 |
| $S_4$ | **10** | 6 | 7 | 2 | 30 |
| Demand | 110 | 200 | 100 | 120 | |

After the first iteration, we have the transportation table given in Table 3. The maximum cost in this table is 8 in cell ($S_1$, $D_3$). Thus, we select row S1 and column $D_3$. The minimum transportation cost is 1 in (S3, D3) at column $D_3$. The maximum assignment to be made for this cell is 30. After this assignment, the capacity of $S_3$ is fully consumed, thus row $S_3$ is removed from the table.

Table 3. Transportation table for the second iteration.

| | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|
| $S_1$ | 3 | 8 | 3 | 185 |
| $S_2$ | 4 | 6 | 3 | 175 |
| $S_3$ | 1 | **1** [30] | 4 | 30 |
| $S_4$ | 6 | 7 | 2 | 30 |
| Demand | 200 | 100 | 120 | |

Since the number of rows or columns is more than one, we start the third iteration. The maximum cost is 8 in cell ($S_1$, $D_3$) in Table 4, we select row $S_1$ and column $D_3$. There are two cells ($S_1$, $D_2$) and ($S_1$, $D_4$) in row $S_1$ that have a minimum cost of 3. In this case, we can choose cell ($S_1$, $D_2$) since the maximum cost in column $D_2$ (6) is bigger than that of column $D_4$ (3) (as an alternative, any minimum cost can be selected randomly, for the sake of simplicity). The maximum assignment for this cell is 185. Thus, we remove row $S_1$ from the table.

Table 4. Transportation table for the third iteration.

| | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|
| $S_1$ | **3** [185] | **8** | **3** | 185 |
| $S_2$ | 4 | 6 | 3 | 175 |
| $S_4$ | 6 | 7 | 2 | 30 |
| Demand | 200 | 70 | 120 | |

Since the number of rows or columns is more than one, we start the fourth iteration. In Table 5, the maximum cost is 7 in cell ($S_4$, $D_3$). Therefore, row $S_4$ and column $D_3$ are selected. The minimum cost in this row and column is 2 in cell ($S_4$, $D_4$). A maximum of 30 can be assigned to this cell. Thus, row $S_4$ is removed from the table.

Table 5. Transportation table for the fourth iteration.

| | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|
| $S_2$ | [4] | [6] | [3] | 175 |
| $S_4$ | [6] | [**7**] | [**2**] 30 | 30 |
| Demand | 15 | 70 | 120 | |

Since there is one remaining row in Table 6, all the necessary assignments are made.

Table 6. Transportation table for the fifth iteration.

| | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|
| $S_2$ | [4] 15 | [6] 70 | [3] 90 | 175 |
| Demand | 15 | 70 | 90 | |

Table 7 gives the final transportation table. The total cost obtained by the AMCM method is 1505, which is also the optimal solution for this problem.

Table 7. Final transportation table.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|---|
| $S_1$ | [10] | [3] 185 | [8] | [3] | 185 |
| $S_2$ | [6] | [4] 15 | [6] 70 | [3] 90 | 175 |
| $S_3$ | [1] 110 | [1] | [1] 30 | [4] | 140 |
| $S_4$ | [10] | [6] | [7] | [2] 30 | 30 |
| Demand | 110 | 200 | 100 | 120 | |

As you can see, this method is very simple, does not involve any calculation, and can be implemented easily in computer programs.

## 6 Experimental study

To show the performance of our method, we compared our method with the well-known methods in the literature which are usually used to test the new methods. 35 problems were used as a benchmark. The number of suppliers and customers and optimal values of the benchmark problems are given in Table 8. Some problems are taken from the literature. We also generated some test problems randomly to see the performance of our method, especially on the larger problems.

The methods used in the comparison are NWC (North-West Corner), LCM (Least Cost Method), RAM (Russel Approximation Method), VAM (Vogel's Approximation Method), RM (Row Minima), CM (Column Minima), TCM (Tuncay Can's Method), and TOCM-SUM (Total Opportunity Cost Matrix-SUM). All the methods used in the comparison were coded in MATLAB, and the experiments were run on a Windows-based PC with a 2.80 GHz Intel Dual Core and 16 GB RAM. Also, the optimal solutions to the test problems were obtained by using Microsoft Excel Solver.

Table 9 gives the number of optimal solutions and the initial basic feasible solutions obtained from each method. First, we compared the methods in terms of the number of optimal solutions obtained.

Table 8. Test problems.

| Problem | Data Source | Number of Customers | Number of Suppliers | Optimal Value |
|---|---|---|---|---|
| PR01 | Random | 6 | 4 | 430 |
| PR02 | Random | 4 | 3 | 12075 |
| PR03 | Random | 4 | 3 | 4010 |
| PR04 | [30] | 5 | 5 | 1102 |
| PR05 | [31] | 4 | 3 | 2850 |
| PR06 | [31] | 4 | 3 | 3320 |
| PR07 | Random | 4 | 4 | 410 |
| PR08 | [31] | 3 | 3 | 1390 |
| PR09 | Random | 4 | 3 | 3100 |
| PR10 | Random | 3 | 3 | 820 |
| PR11 | [32] | 4 | 3 | 167000 |
| PR12 | [33] | 3 | 3 | 1763 |
| PR13 | [32] | 3 | 3 | 1695 |
| PR14 | | 3 | 3 | 1669 |
| PR15 | | 3 | 3 | 1515 |
| PR16 | | 3 | 3 | 530 |
| PR17 | Random | 4 | 3 | 3400 |
| PR18 | Random | 3 | 3 | 129 |
| PR19 | Random | 4 | 3 | 5300 |
| PR20 | Random | 5 | 4 | 204 |
| PR21 | Random | 3 | 3 | 830 |
| PR22 | Random | 3 | 3 | 820 |
| PR23 | Random | 4 | 3 | 6798 |
| PR24 | [34] | 6 | 4 | 71 |
| PR25 | Random | 3 | 3 | 710 |
| PR26 | Random | 81 | 101 | 8399 |
| PR27 | Random | 151 | 201 | 10725 |
| PR28 | Random | 201 | 301 | 16702 |
| PR29 | Random | 351 | 401 | 21807 |
| PR30 | Random | 476 | 501 | 24139 |
| PR31 | Random | 501 | 601 | 29647 |
| PR32 | Random | 676 | 701 | 35935 |
| PR33 | Random | 801 | 801 | 40322 |
| PR34 | Random | 891 | 901 | 44830 |
| PR 35 | Random | 1001 | 1001 | 51204 |

Table 9. The initial solution of the test problems.

| Problem | Optimal | NWC | LCM | RAM | VAM | RM | CM | TCM | TOCM-SUM | AMCM |
|---|---|---|---|---|---|---|---|---|---|---|
| PR01 | **430** | 740 | 450 | 460 | 450 | 490 | 480 | 680 | **430** | 480 |
| PR02 | **12075** | 12200 | 12825 | **12075** | **12075** | 13175 | **12075** | 16825 | 12200 | 12200 |
| PR03 | **4010** | 6580 | **4010** | **4010** | **4010** | **4010** | **4010** | 6880 | **4010** | **4010** |
| PR04 | **1102** | 1994 | 1123 | 1104 | 1104 | 1123 | 1491 | 1927 | 1127 | 1123 |
| PR05 | **2850** | 4400 | 2850 | 2900 | **2850** | **2850** | 3600 | 5350 | **2850** | **2850** |
| PR06 | **3320** | 4160 | **3320** | 3520 | **3320** | **3320** | **3320** | 4320 | **3320** | **3320** |
| PR07 | **410** | 540 | 435 | 440 | 470 | 470 | 435 | 470 | 455 | 435 |
| PR08 | **1390** | 1500 | 1450 | **1390** | 1500 | 1450 | 1500 | 1720 | 1440 | 1440 |
| PR09 | **3100** | 6050 | **3100** | **3100** | **3100** | **3100** | 3200 | 6400 | **3100** | **3100** |
| PR10 | **820** | **820** | 855 | **820** | **820** | 855 | **820** | **820** | **820** | **820** |
| PR11 | **167000** | 193000 | 180500 | 180500 | **167000** | 186000 | 188500 | 184750 | 170250 | **167000** |
| PR12 | **1763** | 1858 | 1832 | 1786 | 1801 | 1822 | 1832 | 1786 | **1763** | **1763** |
| PR13 | **1695** | 1786 | 1784 | 1744 | 1731 | 1774 | 1760 | 1738 | 1696 | 1696 |
| PR14 | **1669** | 1766 | 1752 | 1698 | 1705 | 1728 | 1752 | 1706 | **1669** | **1669** |
| PR15 | **1515** | 1615 | 1715 | 1615 | **1515** | 1545 | 1685 | 1695 | 1565 | 1565 |
| PR16 | **530** | 560 | 555 | **530** | **530** | 560 | 555 | **530** | **530** | **530** |
| PR17 | **3400** | 4750 | 3550 | 3550 | **3400** | **3400** | 4650 | 5850 | **3400** | 3550 |
| PR18 | **129** | 153 | 137 | 133 | **129** | 153 | 137 | 185 | 133 | 141 |
| PR19 | **5300** | 6700 | 6700 | 6100 | **5300** | 6000 | 6000 | 6300 | **5300** | 6100 |
| PR20 | **204** | 358 | **204** | 210 | **204** | **204** | 238 | 296 | 226 | **204** |
| PR21 | **830** | 855 | 830 | 830 | **830** | **830** | 855 | 935 | 910 | 935 |
| PR22 | **820** | **820** | 855 | **820** | **820** | 855 | **820** | **820** | **820** | **820** |
| PR23 | **6798** | 8580 | 6826 | 6826 | **6798** | 6798 | 6826 | 13991 | 6826 | **6798** |
| PR24 | **71** | 109 | 85 | 85 | 77 | 83 | 95 | 113 | **71** | 89 |
| PR25 | **710** | 915 | 735 | **710** | **710** | **710** | 735 | 800 | **710** | 735 |
| PR26 | **8399** | 112865 | 13133 | 13889 | 12379 | 15370 | 13976 | 106830 | 12919 | 15644 |
| PR27 | **10725** | 220879 | 17488 | 18003 | 13719 | 19591 | 20329 | 215897 | 17965 | 17382 |
| PR28 | **16702** | 397551 | 26883 | 28607 | 22914 | 30580 | 27732 | 395520 | 27269 | 27207 |
| PR29 | **21807** | 570544 | 30158 | 31782 | 27731 | 31476 | 32216 | 566884 | 29811 | 29211 |
| PR30 | **24139** | 599718 | 34631 | 34523 | 32365 | 37489 | 34068 | 597255 | 32395 | 34777 |
| PR31 | **29647** | 751815 | 40162 | 41382 | 37689 | 38508 | 42291 | 753766 | 40042 | 39742 |
| PR32 | **35935** | 929497 | 45567 | 45004 | 41243 | 45920 | 45654 | 925185 | 41239 | 45331 |
| PR33 | **40322** | 2011607 | 60003 | 59951 | 55716 | 64575 | 61511 | 2004199 | 59146 | 60431 |
| PR34 | **44830** | 2315462 | 66480 | 66310 | 56345 | 67330 | 74464 | 2310881 | 63177 | 68435 |
| PR35 | **51204** | 2631636 | 76139 | 76423 | 65334 | 79753 | 80147 | 2613239 | 70089 | 75139 |
| No. of Opt. Sol. | **35** | **2** | **4** | **8** | **17** | **8** | **5** | **3** | **14** | **12** |

The number of optimal solutions obtained from each method is depicted in Figure 2. AMCM has found 12 optimal solutions and ranked third after VAM and TOCM-SUM. None of the methods has found the optimal solution for the large size test problems as expected.

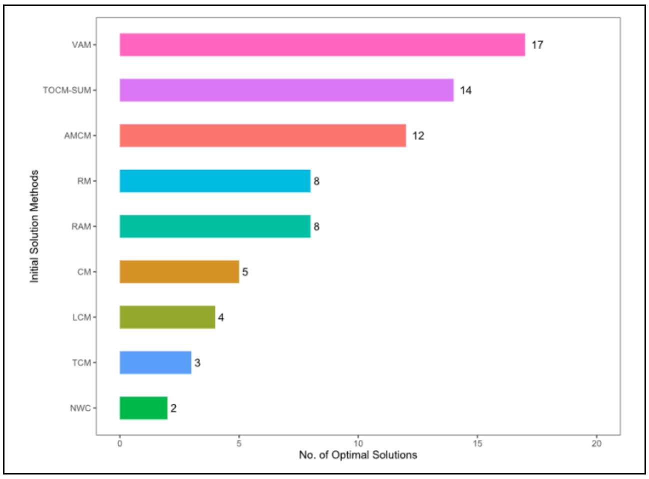


Figure 2. Number of optimal solutions.

Second, we compare the performance of the methods based on the percentage deviations of the IBFS from the optimal solutions. Although it is not expected that an initial solution method to produce an optimal solution, however finding solutions close to the optimal solution will shorten the computation time performed in the second stage to find the optimal solution. The percentage deviations of the test problems obtained from each method are given in Table 10.

Figure 3 shows the mean percentage deviations of the methods. On the average, VAM has the minimum percentage deviation, followed by TOCM-SUM. AMCM, RAM, and LCM have yielded very similar results and they can be ranked as a third category.

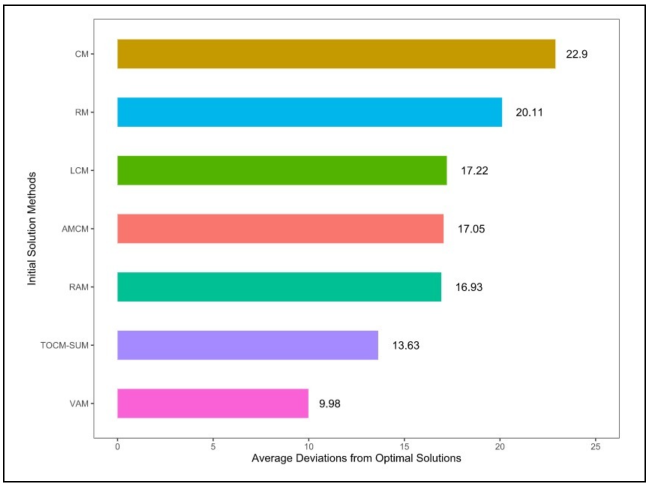


Figure 3. Mean percentage deviation from the optimal solution.

We also calculated some descriptive statistics to see the consistency of our proposed method. Table 11 gives the minimum and maximum deviations, median, first quartiles (Q1), and third quartiles (Q3), and the ranges of the percentage deviations for each method and Figure 4 shows the box plots of the methods. From Table 11 and Figure 4, we can conclude that AMCM has produced very consistent results and is ranked in third place after VAM and TOCM-SUM.

Table 10. Percentage deviation from the optimal solutions

| Problem | NWC | LCM | RAM | VAM | RM | CM | TCM | TOCM-SUM | AMCM |
|---|---|---|---|---|---|---|---|---|---|
| PR01 | 72.09 | 4.65 | 6.98 | 4.65 | 13.95 | 11.63 | 58,14 | **0,00** | 11.63 |
| PR02 | 1.04 | 6.21 | **0.00** | **0.00** | 9.11 | **0.00** | 39,34 | 1,04 | 1.04 |
| PR03 | 64.09 | **0.00** | **0.00** | **0.00** | **0.00** | **0.00** | 71,57 | **0,00** | **0.00** |
| PR04 | 80.94 | 1.91 | 0.18 | 0.18 | 1.91 | 35.30 | 74,86 | 2,27 | 1.91 |
| PR05 | 54.39 | **0.00** | 1.75 | **0.00** | **0.00** | 26.32 | 87,72 | **0,00** | **0.00** |
| PR06 | 25.30 | **0.00** | 6.02 | **0.00** | **0.00** | **0.00** | 30,12 | **0,00** | **0.00** |
| PR07 | 31.71 | 6.10 | 7.32 | 14.63 | 14.63 | 6.10 | 14,63 | 10,98 | 6.10 |
| PR08 | 7.91 | 4.32 | **0.00** | 7.91 | 4.32 | 7.91 | 23,74 | 3,60 | 3.60 |
| PR09 | 95.16 | **0.00** | **0.00** | 0.00 | **0.00** | 3.23 | 106,45 | **0,00** | **0.00** |
| PR10 | **0.00** | 4.27 | **0.00** | 0.00 | 4.27 | **0.00** | **0,00** | **0,00** | **0.00** |
| PR11 | 15.57 | 8.08 | 8.08 | 0.00 | 11.38 | 12.87 | 10,63 | 1,95 | **0.00** |
| PR12 | 5.39 | 3.91 | 1.30 | 2.16 | 3.35 | 3.91 | 1,30 | **0,00** | **0.00** |
| PR13 | 5.37 | 5.25 | 2.89 | 2.12 | 4.66 | 3.83 | 2,54 | 0,06 | 0.06 |
| PR14 | 5.81 | 4.97 | 1.74 | 2.16 | 3.54 | 4.97 | 2,22 | **0,00** | **0.00** |
| PR15 | 6.60 | 13.20 | 6.60 | **0.00** | 1.98 | 11.22 | 11,88 | 3,30 | 3.30 |
| PR16 | 5.66 | 4.72 | **0.00** | **0.00** | 5.66 | 4.72 | **0,00** | **0,00** | **0.00** |
| PR17 | 39.71 | 4.41 | 4.41 | **0.00** | **0.00** | 36.76 | 72,06 | **0,00** | 4.41 |
| PR18 | 18.60 | 6.20 | 3.10 | **0.00** | 18.60 | 6.20 | 43,41 | 3,10 | 9.30 |
| PR19 | 26.42 | 26.42 | 15.09 | **0.00** | 13.21 | 13.21 | 18,87 | **0,00** | 15.09 |
| PR20 | 75.49 | **0.00** | 2.94 | **0.00** | **0.00** | 16.67 | 45,10 | 10,78 | 0.00 |
| PR21 | 3.01 | **0.00** | **0.00** | **0.00** | **0.00** | 3.01 | 12,65 | 9,64 | 12.65 |
| PR22 | **0.00** | 4.27 | **0.00** | **0.00** | 4.27 | **0.00** | **0,00** | **0,00** | **0.00** |
| PR23 | 26.21 | 0.41 | 0.41 | **0.00** | **0.00** | 0.41 | 105,81 | 0,41 | **0.00** |
| PR24 | 53.52 | 19.72 | 19.72 | 8.45 | 16.90 | 33.80 | 59,15 | **0,00** | 25.35 |
| PR25 | 28.87 | 3.52 | **0.00** | **0.00** | **0.00** | 3.52 | 12,68 | **0,00** | 3.52 |
| PR26 | 1243,79 | 56,36 | 65,36 | 47,39 | 83,00 | 66,40 | 1171,94 | 53,82 | 86,26 |
| PR27 | 1959,48 | 63,06 | 67,86 | 27,92 | 82,67 | 89,55 | 1913,03 | 67,51 | 62,07 |
| PR28 | 2280,26 | 60,96 | 71,28 | 37,19 | 83,09 | 66,04 | 2268,10 | 63,27 | 62,90 |
| PR29 | 2516,33 | 38,30 | 45,74 | 27,17 | 44,34 | 47,73 | 2499,55 | 36,70 | 33,95 |
| PR30 | 2384,44 | 43,46 | 43,02 | 34,08 | 55,30 | 41,13 | 2374,23 | 34,20 | 44,07 |
| PR31 | 2435,89 | 35,47 | 39,58 | 27,13 | 29,89 | 42,65 | 2442,47 | 35,06 | 34,05 |
| PR32 | 2486,61 | 26,80 | 25,24 | 14,77 | 27,79 | 27,05 | 2474,61 | 14,76 | 26,15 |
| PR33 | 4888,86 | 48,81 | 48,68 | 38,18 | 60,15 | 52,55 | 4870,49 | 46,68 | 49,87 |
| PR34 | 5064,98 | 48,29 | 47,91 | 25,69 | 50,19 | 66,10 | 5054,76 | 40,93 | 52,65 |
| PR35 | 5039,51 | 48,70 | 49,25 | 27,60 | 55,76 | 56,52 | 5003,58 | 36,88 | 46,74 |
| **Mean Dev.** | **887,11** | **17,22** | **16,93** | **9,98** | **20,11** | **22,90** | **885,08** | **13,63** | **17,05** |

Table 11. Descriptive statistics of the results

| Methods | Min | Q1 | Median | Q3 | Max |
|---|---|---|---|---|---|
| NWC | 0,00 | 7,26 | 39,71 | 1601,63 | 5064,98 |
| LCM | 0,00 | 3,72 | 5,25 | 31,14 | 63,06 |
| RAM | 0,00 | 0,09 | 4,41 | 32,41 | 71,28 |
| VAM | 0,00 | 0,00 | 0,18 | 20,23 | 47,39 |
| RM | 0,00 | 0,95 | 5,66 | 28,84 | 83,09 |
| CM | 0,00 | 3,68 | 11,63 | 38,95 | 89,55 |
| TCM | 0,00 | 12,66 | 58,14 | 1542,48 | 5054,76 |
| TOCM-SUM | 0,00 | 0,00 | 1,95 | 24,48 | 67,51 |
| AMCM | 0,00 | 0,00 | 3,60 | 30,05 | 86,26 |

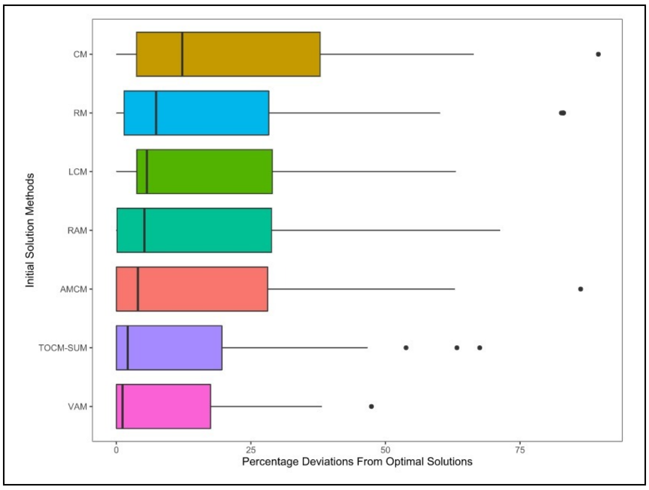


Figure 4. Percentage deviation from the optimal solution.

## 7 Discussions and conclusion

The transportation problem is one of the most important problems in optimization. This simple but important problem has so many applications in real life and is also considered one of the basic problems in the operation research courses to give students the essence of optimization.

The transportation problem is solved in two phases. First, an initial basic solution is found, later this solution is improved until an optimal solution is obtained. Therefore, finding a good initial solution would shorten the number of steps in the second phase. In this study, a new method is proposed to find an initial basic solution for the transportation problem. The idea behind the method is to avoid the largest costs as much as possible. To achieve this, the highest priority is given to rows or columns, in the transportation table that contains the largest costs, and assignments are made to the cells that have the smallest cost in respected rows or columns. This method usually yields very good initial basic solutions and even optimal solutions for some problems. We compared the performance of the method on thirty-five test problems with well-known methods, and obtain very good and consistent results.

The advantage of this method is that it requires simple comparisons compared to some methods that require some calculations such as finding the difference between two costs, calculating averages, etc. Therefore, this method is very simple. Because of its simplicity, it can be used especially in the lectures as an alternative method for finding an initial basic solution in transportation problems besides well-known methods such as NWC, LCM, and VAM.

The proposed method may be improved by modifications of the algorithm or combining it with other methods such as VAM. These improvements will be investigated in further studies.

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## 9 Author contributions

The authors confirm that contributions to the paper made by authors as follows: study conception and development of the method by Özcan MUTLU, data collection by Kenan KARAGÜL and Yusuf ŞAHİN; analysis and interpretation of results by Özcan MUTLU, Kenan KARAGÜl, and Yusuf ŞAHİN; draft manuscript preparation by Özcan MUTLU, Kenan KARAGÜL, and Yusuf ŞAHİN.

## 10 Ethics committee approval and conflict of interest

There is no need to obtain ethics committee approval in the prepared article.

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