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STRESS ANALYSIS IN A ROTATING DISC WITH CONSTANT SURFACE VELOCITY

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ABSTRACT

This study is concerned with the stress analysis of a rotating disc with a constant surface velocity. It is assumed that the mass of the disc is variable and is a function of time. Decrease in angular velocity causes angular acceleration and, therefore, shear stress field. Differential equations of motion for this case are solved analytically and shown that the present study is a generalization of the problem of a disc rotating with constant angular velocity. Stresses occurring in the problem are obtained for different values of surface velocity.

Key Words : Stress analysis, Rotating disc, Angular acceleration, Shear stress

SABİT YÜZEY HIZIYLA DÖNEN DİSKTE GERİLME ANALİZİ

ÖZET

Bu çalışma sabit yüzey hızıyla dönen bir diskteki gerilme analizi ile ilgilidir. Diskin kütlesinin değişken ve zamanın bir fonksiyonu olduğu farzedilmektedir. Açısal hızdaki azalma ivmeye dolayısıyla da kayma gerilmesi alanının ortaya çıkmasına sebebiyet verir. Bu hale ait hareket denklemleri analitik olarak çözülmekte ve mevcut çalışmanın sabit açısal hızda dönen disk probleminin bir genelleştirilmesi olduğu gösterilmektedir. Problemde ortaya çıkan gerilmeler, yüzey hızının farklı değerleri için elde edilmektedir.

Anahtar Kelimeler : Gerilme analizi, Dönen disk, Açısal ivme, Kayma gerilmesi

1. INTRODUCTION

A disc with variable mass represents a fundamental working element of large number of textile machines (Cheviticanin, 1988). Mass and geometry are varying in the course of running period of the machine. Although the disc with constant angular velocity has been studied extensively, the problem of disc with variable mass has not been extensively investigated. Therefore, the aim of this paper is to analyze the stress distribution in a disc on which the textile band is wound up with constant velocity. In this case, the winding up is regular and the force in the textile band is constant. But, to have a constant velocity, it is necessary to connect the disc with a variator of velocity. This machinery makes the system more complex, complicated and expensive. Because of that, in every day use, the textile band is wound with constant velocity.

Stress distribution in a disc which rotates with constant angular velocity has been investigated in references (Venkatraman and Sharad, 1970; Alexander and Gunesekara, 1991; Shames, et all., 1992). Since the disc rotates with constant angular velocity, angular acceleration becomes zero and therefore the problem considered is reduced to determining the stresses σ_r and σ_θ . Whereas, in the case that the angular velocity is variable, in addition to the inertial forces in the radial direction, shear stresses must occur since the acceleration field will occur in the tangential direction. With this form, the problem forms a generalization of the disc problem with constant angular velocity.

2. ANALYSIS

The simplified model of a disc is shown in Figure 1.

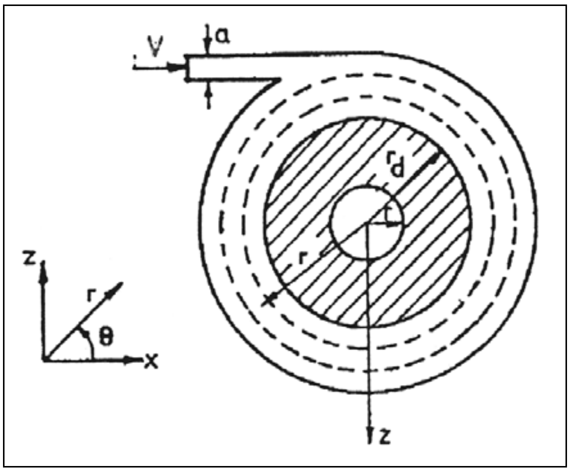

Figure 1. Rotating disc with constant surface velocity

It is assumed here that the shaft on which the disc is mounted is so short that the deflection in the z direction can be omitted. Mass m(t), radius r (t) are varying with time and are functions of the number of winding.

Now, we assume that the textile band is wound up with the constant surface velocity v. Let the initial radius of the disc be r_d . Then, we can write the radius at any time as

$$r = r_d + (n - 1/2) \tag{1}$$

where n is the number of winding. The instantaneous angular velocity will be given by

$$w = \frac{v}{r} = \frac{v}{r_d + (n - 1/2)a} \tag{2}$$

where a, r_d are the thicknesses of the textile band and outer radius of the disc, respectively. The angular acceleration can be found by differentiating Eq. (2):

$$a = -\frac{v}{r^2}\left(\frac{dr}{dt}\right) \tag{3}$$

In order to find the term dr/dt in Eq.(3), we consider a control volume as in Figure2. In the course of time dt, it can be written for the part that is wound up on the disc that

$$V = 2\pi r(-dr) = av(dt) \tag{4}$$

or

$$\frac{dr}{dt} = \frac{-av}{2\pi r} \tag{5}$$

Substituting Eq.(5) into Eq.(3), we have

$$\alpha = \frac{av^2}{2\pi r^3} \tag{6}$$

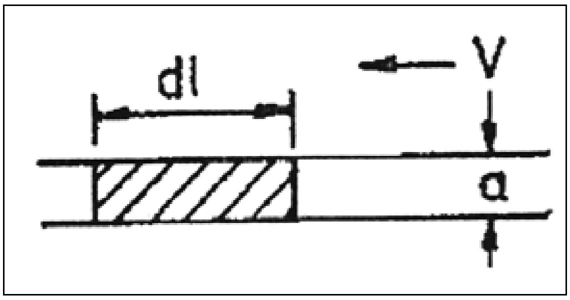

Figure 2. Control volume

2. 1. Stress Analysis

In order to find the stress distribution in the disc occurring because of the radial and circumfrential forces, as different from the case of disc with constant angular velocity, we assume that the stress field will include the shear stress $\tau_{r\theta}$, that is, the stress field will include the stresses σ_r, σ_θ and $\tau_{r\theta}$. Because of the symmetry, we can assume that the stresses are independent of the angle θ. Although there exists a difference of one layer between the lower and upper parts of the disc, this difference is negligibly small and can be neglected. The equation of motion in this case will be

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + \rho_d w^2 r = 0 \tag{7}$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} + \rho_d \frac{av^2}{2\pi r^2} = 0$$

where ρ_d is the density of the disc. On the other hand, strain rates caused by these stresses are given by (Venkatraman, and Sharad, 1970)

$$\varepsilon_r = \frac{du}{dr}, \varepsilon_\theta = \frac{u}{r}, \gamma_{r\theta} = \frac{dv}{dr} - \frac{v}{r} \tag{8}$$

Here, again, the dependence of strain rates upon the angle θ is not considered.

The last five compatibility equations that must be satisfied by Eq.(6) are identically satisfied. The first compatibility equation

$$r\frac{\partial \varepsilon_r}{\partial r} - \frac{\partial^2 \varepsilon_r}{d\theta^2} - \frac{\partial}{\partial r}\left[r\left(r\frac{\partial \varepsilon_\theta}{\partial r} - \frac{\partial \gamma_{r\theta}}{\partial \theta}\right)\right] = 0 \tag{9}$$

gives

$$\frac{d\varepsilon_r}{dr} + \frac{\varepsilon_\theta - \varepsilon_r}{r} = 0 \qquad (10)$$

for this case (Venkatraman and Sharad, 1970)

Now, substitution of stress-strain relations and Eqs. (7) in Eq. (10), and the simplification gives

$$\begin{aligned} & r^2 \frac{d^2\sigma_r}{dr^2} + 3r\frac{d\sigma_r}{dr} \\ & = \frac{1}{r}\frac{d}{dr}\left(r^3 \frac{d\sigma_r}{dr}\right) = -(3+\nu)\rho w^2 r^2 \end{aligned} \qquad (11)$$

The solution of this equation is

$$\sigma_r = \frac{-(3+\nu)}{8}\rho w^2 r^2 - \frac{c_1}{c_2} + c_2 \qquad (12)$$

as is given in standard books on differential equations. Here, c_1 and c_2 are the constants that must be determined by means of the boundary conditions.

The explicit form of $\tau_{r\theta}$ can be found from the second of Eqs.(7). The solution of the homogenous part is $\tau_{r\theta}=c_4/r^2$. Let the proper solution be $\tau_{r\theta}=c_5/r$. Substituting this expression into the second equation in Eqs.(7), we find $c_5= -\rho_d a v^2/2\pi$. Thus, the general solution is in the following form

$$\tau_{r\theta} = \frac{c_4}{r} - \frac{\rho_d a v^2}{2\pi r} \qquad (13)$$

2. 2. Boundary Condition

In order to obtain the boundary condition concerned with σ_r , we remark that this stress will equal to the difference between inertia and weight forces per unit width. The force ΔF_r acting upon the angular element shown in Figure 2 is

$$\begin{aligned} \nabla F_{r_1} &= \rho_m d\theta \int_{r=r_d} \rho_m r\left(w^2 r\right) dr d\theta \\ &= \frac{1}{3}\rho_m w^2 \left((r_d + a)^3 - r_d{}^3\right) d\theta \end{aligned} \qquad (14)$$

where ρ_m is the density of the textile material. On the other hand, the mass force in the radial direction is given by

$$\begin{aligned} \nabla F_{k_1} &= g\rho_m d\theta \int_{r=r_d}^{r=r_d+d} r\cos\theta \;\; dr \\ &= g\rho_m \left[(r_d + a)^2 - r_d{}^2\right]\cos\theta \;\; d\theta \end{aligned} \qquad (15)$$

Here, $d\theta$ is the angle made by the horizontal direction and g is the acceleration of gravity. The difference of these two forces gives the net radial force acting upon the disc surface. Dividing this difference by $r_d d\theta$ we obtain the stress σ_r per unit line element in the radial direction:

$$\begin{aligned} & \sigma_{r_1} = \frac{\rho_m}{r_d}\left[\frac{w^2}{3}\left[(r_d + a)^3 - r_d^3\right] - \right. \\ & \left. g\sin\theta\left[(r_d + a^2) - r_d^2\right]\right] = \Phi_1 \end{aligned} \qquad (16)$$

Here, it will be assumed that there is no separation and sliding between the textile band and the disc surface. Since the angular velocity will decrease with the increase in the number of winding up, this assumption will be valid with the increase in time, and is in agreement with the experimental evidences. Likewise, inertia and mass forces occurring in the second layer are found to be

$$\nabla F_{r2} = \frac{1}{3}\rho_m w^2 \left[(r_d + 2a)^3 - (r_a + a)^3\right] d\theta \qquad (17)$$

and

$$\nabla F_{r_2} = g\rho_m \left[\left[r_d + 2a\right]^2 - (r_d + a)^2 \sin\theta d\theta\right] \qquad (18)$$

Division of the difference of these two forces by $(r_d+a)d\theta$ gives the radial stress σ_r in the second layer:

$$\begin{aligned} & \sigma_{r_2} = \frac{\rho_m}{(r_d + a)}\left[\frac{w^2}{3}\left[(r_d + 2a)^3 - (r_d + a)^3\right] - \right. \\ & \left. g\left[(r_d + 2a)^2 - (r_d + a)^2\right]\sin\theta \right] = \phi_2 \end{aligned} \qquad (19)$$

Continuing in this way, we have for the stress σ_r between the n th. and (n-1) th. windings

$$\begin{aligned} & \sigma_{r_n} = \frac{\rho_m}{\left[r_d + (n-1)a\right]}\left[(r_d + na)^3 - (r_d + (n-1)a)^3\right] \\ & - \sin\theta\left[(r_d + na)^2 - (r_d + (n-1)a)^2\right]^2 = \phi_n \end{aligned} \qquad (20)$$

Now, the resultant radial stress σ_r on the disc surface, which will be taken as the boundary condition depending upon the number of winding up, will be the summation of these n stresses:

$$\sigma_r = \rho_m \sum_{k=1}^{n}\left\{ \frac{1}{\left(r_d + (k-1)a\right)}\left[\frac{w^2}{3}\left[(r_d + ka)^3 - \left(r_d + (k-1)a\right)^3\right]\right]\right.$$
$$\left. - g\sin\theta\left[(r_d + ka)^2 - (r_d + (k-1)a)^2\right]\right\} \tag{21}$$
$$= \phi_1 + \phi_2 + ..\phi_n = \sum_{k=1}^{n}\phi_n = \psi(\rho_m, r_d, a, w, \theta, q)$$

Here, it can readily be shown that the relation between the number of winding and angular velocity is n=wt, wherein t is time parameter.

In the same manner, we will proceed to obtain the boundary value for shear stress. The force occurring on the first winding up due to angular acceleration is

$$\Delta F_{t_1} = \int_{r_d}^{r_d+a} \rho_m r(\alpha r)\,dr\,d\theta = \rho_m d\theta \int_{r_d}^{r_d+a} \frac{av^2}{2\pi r^3} r^2 dr = \frac{a\rho_m v^2 d\theta}{2\pi} \ln \frac{r_d + a}{r_d} \tag{22}$$

Shear forces per unit line element between first and second layers is found to be

$$\Delta F_{t_1} = \int_{r_d}^{r_d+a} \rho_m r(\alpha r)\,dr\,d\theta = \rho_m d\theta \int_{r_d}^{r_d+a} \frac{av^2}{2\pi r^3} r^2 dr = \frac{a\rho_m v^2 d\theta}{2\pi} \ln \frac{r_d + a}{r_d} \tag{23}$$

In the same manner, it is found for the n th and (n-1) th layers that the shear force per unit line element is

$$\Delta F_{t_2} = \frac{a\rho_m v^2}{2\pi r_d} \ln \frac{r_d + a}{r_d} \tag{24}$$

The frictional force per unit line segment between the disc and the winding up is

$$\Delta F_n = \mu\sigma_r \tag{25}$$

where σ_r is given by Eq.(20). Likewise, frictional force between first and second layers, and n th and (n-1) th layers are, respectively, given by

$$\Delta F_n = \mu\sigma_r \tag{26}$$

and

$$\Delta F_{s_n} = \mu_s \sigma_{r_{t_n}}$$
$$= \mu_s \frac{\rho_m}{\left(r_d + (k-1)a\right)}\left[\frac{w^2}{3}\left[(r_d + ka)^3 - \left(r_d + (k-1)a\right)^3\right]\right] \tag{27}$$
$$\left. - g\sin\theta\left[(r_d + ka)^2 - \left(r_d + (k-1)a\right)^2\right]\right\}$$

where σ_r is given as

$$\sigma_{t_2} = \rho_m \sum_{k=2}^{n}\left\{ \frac{1}{\left(r_d + (k-1)a\right)}\left[\frac{w^2}{3}\left[(r_d + ka)^3\right.\right.\right.$$
$$\left.\left. - \left(r_d + (k-1)a\right)^3\right]\right] - \tag{28}$$
$$\left. g\sin\theta\left[(r_d + ka)^2 - \left(r_d + (k-1)a\right)^2\right]\right\}$$

Here, μ_s is the static coefficient of friction. Again, we have accepted in deriving the above equations that there is no sliding among the layers, and this fact is supported by experimental evidences.
In calculating the forces in the longitudinal direction, it is necessary to include the projection of these forces in that direction. Thus, the mass on the first layer between the angles θ and θ+dθ is

$$\frac{\rho_m g \cos\theta d\theta}{2}\left[(r_d + a)^2 - r_d^{\,2}\right] = \varphi_1 \tag{29}$$

Then, the mass per unit line segment is

$$\frac{\rho_m g \cos\theta d\theta}{2}\left[(r_d + a)^2 - r_d^{\,2}\right] = \varphi_1 \tag{30}$$

The weights on the second and n th layers are

$$\frac{\rho_m \cos\theta}{2r_d}\left((r_d + a)^2 - r_d^2\right) \tag{31}$$

and

$$\frac{g\rho_m \cos\theta}{2\left(r_d + (n-1)a\right)}\left[(r_d + na)^2 - \left(r_d + (n-1)a\right)^2\right] = \varphi_n \tag{32}$$

The expressions (28), (29), (31) and (32) will be used in calculating the shear stress $\tau_{r\theta}$.

Since we assume that there is no sliding between layers, the shear stress on the surface of the disc will be the summation of the frictional forces and the components of the weights of layers in the longitudinal direction:

$$\tau_{r\theta} = \sum_{k=1}^{n}\Delta F_{t_k} + \sum_{k=1}^{n}\varphi_1 - \mu\sigma_r \tag{33}$$

or, in the explicit form,

$$\tau_{r\theta} = \frac{a\rho_m v^2}{2\pi}\left\{\sum_{k=1}^{n}\frac{1}{\left[r_d + (k-1)a\right]}\ln\frac{r_d + ka}{r_d + (k-1)a}\right\}$$
$$+ \frac{g\rho_m r_d \cos\theta}{2}\sum_{k=1}^{n}\frac{1}{\left[r_d + (k-1)a\right]}\left[(1 + k(a/r_d))^2 - (1 + (k-1)(a/r_d))^2\right]$$

$$-\mu\rho_m \sum_{k=1}^{n}\left\{\frac{1}{r_d+(k-1)a}\left[\frac{w^2}{3}\left[(r_d+ka)^3-(r_d+(k-1)^3)\right]\right\}\right. \tag{34}$$

$$\left.-g\sin\theta\left[(r_d+ka)^2-(r_d+(k-1)a)^2\right]\right]$$

Now, we can find the constants in Eq.(12) and Eq.(13) by means of these boundary values obtained. Since Eq.(12) gives $\sigma_r=\infty$ at the point r = 0, c_3 must be zero: c_3=0. On the other hand, by Eq.(21), since $\psi=\sigma_r$ on the boundary surface, we have

$$\psi=\frac{-3+\mu}{8}\rho w^2 r_d^2+c_4 \tag{35}$$

or

$$C_4=\Psi+\frac{3+v}{8}\rho w^2 r_d^2 \tag{36}$$

Substituting Eq.(30) into the expression for σ_r and making the necessary changes, we find

$$\sigma_r=\psi+\frac{3+\mu}{8}\rho w^2 r_d^2\left[1-\left(\frac{r}{r_d}\right)^2\right] \tag{37}$$

The longitudinal stress σ_θ is obtained by substituting Eq.(37) into the first of Eqs.(7):

$$\sigma_r=\psi+\frac{3+\mu}{8}\left[1-\left(\frac{1+3\mu}{3+\mu}\right)\left(\frac{r}{r_d}\right)^2\right]\rho w^2 r_d^2 \tag{38}$$

In order to find $\tau_{r\theta}$ we remind that $\tau_{r\theta}=\psi_d$ on the disc surface. Utilizing this condition, we have for the constant c_4 in Eq. (34)

$$c_4=r_d^2\psi_d+\frac{\rho_d a v^2 r_d}{2\pi} \tag{39}$$

Substituting this expression into the expression for $\tau_{r\theta}$ we obtain

$$\tau_{r\theta}=\psi_d\left(\frac{r}{r_d}\right)^2+\frac{\rho_d a v^2 r_d}{2\pi r}\left(\frac{r}{r_d}-1\right) \tag{40}$$

When we look at this equation carefully, we can see that it gives the stress value as infinity at the center. In this case , it must be expected that Eq.(40) can give results in satisfactory agreement with the experimental results near disc center since the expression (13) contains only one constant.

2. 3. Displacement Field

In order to find displacement components u,v and $\gamma_{r\theta}$ due to σ_r, σ_θ and $\tau_{r\theta}$ stress-strain relations (Venkatraman and Sharad, 1970) must be utilized. Taking the first equation in (8) into account and substituting Eq. (37) and Eq.(38) into the expression for radial strain rate ε_r in the stress-strain relations, we have

$$u=\frac{(3+\nu)(1-\nu)(r/r_d)r_d}{8E}\cdot\left[\psi+\left[1-\left(\frac{1+3\mu}{3+\mu}\right)\left(\frac{r}{r_d}\right)^2\right]\right]\rho_d w^2 r_d^2 \tag{41}$$

In the same manner, calculating the same equality $\tau_{r\theta}=G\gamma_{r\theta}$ with the help of third one of Eqs.(8) and substituting the resulting equation into the second one in Eqs.(7), we arrive at the equation

$$r^2\frac{d^2v}{dr^2}+r\frac{dv}{dr}-v+\frac{\rho a v^2}{2\pi G}=0 \tag{42}$$

from which it can easily be found that the solution is

$$v=\frac{c_1}{r}+c_2 r+c_0 \tag{43}$$

It must be seen that the term c_2r in the expression corresponds to the rigid body motion and does not contribute to deformation. Thus, c_2 must be zero. On the other hand, since it must be necessary that v=0 at r = 0. c_1 must also be zero. Thus, Eq. (43) gives v=c_0. c_0 can be found from the condition $\tau_{r\theta}=\psi_d$

$$\tau_{r\theta}=G\left(\frac{dv}{dt}-\frac{v}{r}\right)=-g\frac{v}{r} \tag{44}$$

so as to have

$$c_0=\frac{-\psi r_d}{G} \tag{45}$$

Thus, using Eq.(45), we finally have from Eq. (43) and Eq.(44)

$$\tau_{r\theta}=\psi_d\left(\frac{r_d}{r}\right),\qquad v=-\psi_d\frac{r_d}{G},\quad 0\le r\le r_d \tag{46}$$

We see that Eq.(46) so obtained and Eq.(40) are different from each other. The difference between these two equations is that some limitations concerned with displacements are put in finding the second one. It must be expected that the second expression will give smaller values and therefore more expectable results compared to those obtained by means of the first equation for $\tau_{r\theta}$.

2. 4. The Annular Disc Case

For an annular disc, the constant c_1 does not have to be zero. But, in this case, the boundary condition $\sigma_r(r_i)=\tau_{r\theta}(r_i)=0$, $\sigma_r(r_d)=\psi$ and $\tau_{r\theta}(r_d)=\psi_d$ must be utilized. Using Eq.(40) and the last of Eqs.(7), it is found from the condition $\tau_{r\theta}(r_i)=0$ that

$$c_1 = \frac{1}{2G}\left(\frac{\psi_d r_d^2 r_i^2}{\left(r_d^2 - r_i^2\right)}\right), c_0 = \frac{1}{G}\left(\frac{\psi_d r_d^2}{r_d^2 - r_i^2}\right) \tag{47}$$

Substituting these constants into the expression for $\tau_{r\theta}$, we have

$$\tau_{r\theta} = \frac{\psi_d r_d^2}{r_i^2 - r_d^2}\left[\left(\frac{r_i}{r}\right)^2 - 1\right] \tag{48}$$

In a like manner, using the boundary conditions $\sigma_r(r_i)= \sigma_r(r_d)=0$ in Eq.(12), we have for two constants

$$c_1 = -\left[\psi + \frac{3+\nu}{8}\left(r_d^2 - r_i^2\right)\rho w^2\right]\frac{2r_d^2 r_i^2}{r_i^2 - r_d^2}$$

$$c_2 = \frac{3+\nu}{8}\rho w^2 r_i^2 \tag{49}$$

$$-\frac{r_d^2}{\left(r_i^2 - r_d^2\right)}\left[\psi + \frac{3+\nu}{8}\rho w^2\left(r_d^2 - r_i^2\right)\right]$$

Substituting these expressions into Eq.(12), we obtain

$$\sigma_\theta = \frac{3+\nu}{8}\left[\left(1+\frac{r_i^2}{r_d^2}\right) - \frac{r_i^2/r_d^2}{(r/r_d)^2} - \left(\frac{1+3\nu}{3+\nu}\right)\left(\frac{r}{r_d}\right)^2\right]\rho w^2 r_d^2$$

$$+\frac{\psi r_d^2}{\left(r_d^2 - r_i^2\right)}\left[1 + \frac{\left(r_d / r_d\right)^2}{(r / r_d)^2}\right] \tag{50}$$

Now, substituting Eq.(50) into the first of Eqs.(7), we have for $\sigma_{r\theta}$

$$\sigma_\theta = \frac{3+\nu}{8}\left[\left(1+\frac{r_i^2}{r_d^2}\right) - \frac{r_i^2/r_d^2}{(r/r_d)^2} - \left(\frac{1+3\nu}{3+\nu}\right)\left(\frac{r}{r_d}\right)^2\right]\rho w^2 r_d^2$$

$$+\frac{\psi r_d^2}{\left(r_d^2 - r_i^2\right)}\left[1 + \frac{\left(r_i / r_d\right)^2}{(r / r_d)^2}\right] \tag{51}$$

Displacement component u can be found by replacing Eq.(49) and Eq.(50) in the stress-strain relations:

$$u = \frac{(3+\nu)(1-\nu)(r/r_d)}{8E}\left[\left(1+\frac{r_i^2}{r_d^2}\right) - \frac{1+\nu}{1-\nu}\frac{r_i^2/r_d^2}{(r/r_d)^2}\right.$$

$$\left. -\left(\frac{1+3\nu}{3+\nu}\right)\left(\frac{r}{r_d}\right)\rho\right] w^2 r_d^2$$

$$+\frac{\psi r_d^2 (r/r_d) r_d}{\left(r_d^2 - r_i^2\right)E}\left[1 - \frac{\left(r_i / r_d\right)^2}{(r / r_d)^2}\right] \tag{52}$$

In order to find the displacement component v, we remind that the shear stress $\tau_{r\theta}$ can be written by means of Eq.(44) as

$$\tau_{r\theta} = \left(\frac{-2c_1}{r^2} - \frac{c_0}{r}\right)G \tag{53}$$

Using the boundary conditions $\tau_{r\theta}(r = r_i) = 0$ and $\tau_{r\theta}(r = r_d) = \psi_d$, we find

$$c_0 = \frac{2r_i r_d^2 \psi_d}{1 - 2r_i r_d}, c_1 = -\frac{\psi_d r_d^2}{1 - 2r_i r_d} \tag{54}$$

Finally, substituting these expressions into Eqs.(44) and (53), we obtain

$$\tau_{r\theta} = \frac{2r_d^2 \psi_d G}{(1 - 2r_i r_d)}\left[\frac{1}{r} - r_i\right] \tag{55}$$

$$v = \frac{2r_d^2 \psi_d G}{(1 - 2r_i r_d)}\left[r_i - \frac{1}{r}\right]$$

3. RESULTS

In this study, we have dealt with the stress analysis in a disc on which the textile band is wound with a constant surface velocity and shown that the shear stress must also occur in the disc, as different from the case of rotating disc with a constant angular velocity. In the calculations, stresses on the disc surface due to centrifugal and gravitational forces have also been taken into account. With this form, the present analysis is a generalization of the rotating disc with constant angular velocity. Indeed, it can easily be seen that Eqs.(37), (38), (41), (50) and (52) give the results for rotating disc with constant angular velocity if the boundary values on the disc surface arising from the winding of textile material are taken as zero. Since the shear stress gives infinity at the disc center for a solid disc , two formulas have been developed. Although both formulas give infinity at r=0, the second one gives smaller values near the center and it must be expected that this formula gives results in agreement with the experimental ones for the values bigger than $r/r_d \geq 0.1$, as is clear from Figure3.

The stresses σ_r and σ_θ belonging to annular disc are shown in Figure 3 for different values of the surface velocity.

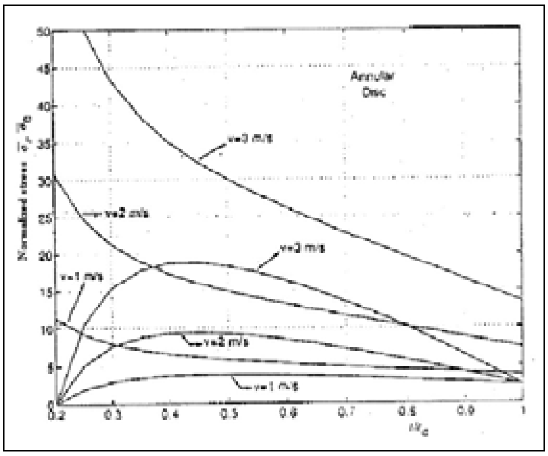

Figure 3. The variation of radial and hook stresses $\bar{\sigma}_r$ and $\bar{\sigma}_\theta$ with r/r_d

Here, the ratio $r_i/r_d = 1/5$ and v = 0.3 are taken and $\bar{\sigma}_r$ and $\bar{\sigma}_\theta$ show σ_r/E and σ_θ/E, respectively. Figure3 shows the variation of σ_r and σ_θ wiht r/r_d . As is seen from this figure, σ_r takes its maximum value at the point $x = 0.4(r/r_d)$ while $\bar{\sigma}_\theta$ decreases with r/r_d. In Figure 4, the variation of σ_r with the number of winding for a sold disc is shown so as to have an idea on the effect of the number of winding, and it is seen that the stress σ_r decreases with the increase in the number of winding.

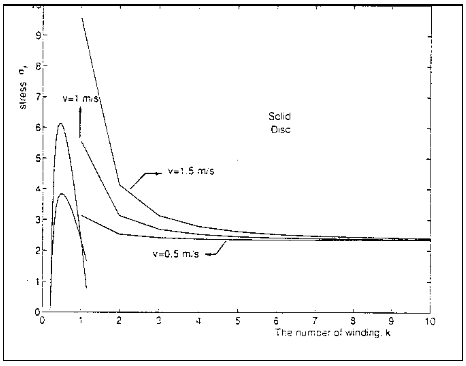

Figure 4. The variation of radial stress σ_r with the number of winding for a solid disc

Figure 5 shows the variation of displacement u corresponding to σ_r with the ratio r/r_d . Dotted lines belong to annular disc with $r_i/r_d = 1/5$ while solid lines show results for a solid disc.

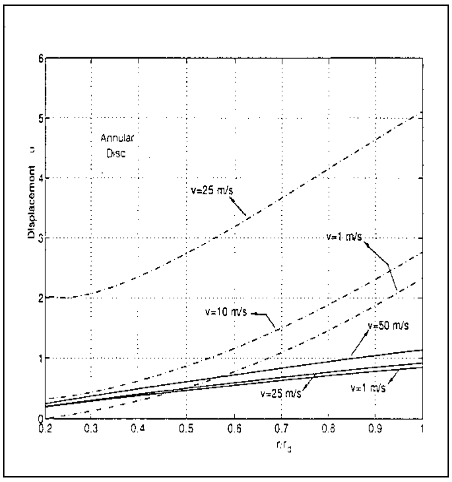

Figure 5. The variation of Displacement u with r/r_d

We see from these curves that the displacement u does not excessively depend on the surface velocity. This is especially clear in the case of solid disc.

In order to visualize the difference between formulas (40) and (46) for shear stress, Figure 6 has been plotted. It can clearly be seen that the values obtained by using Eq.(46) is smaller than those obtained from Eq.(40) near the disc center in the case of solid disc.

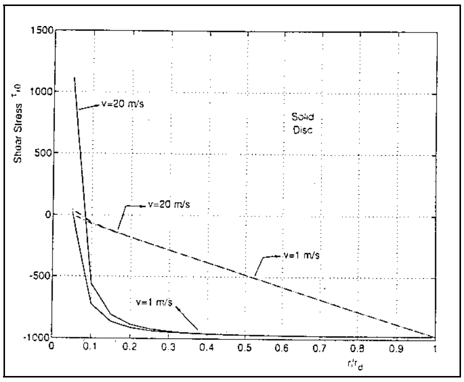

Figure 6. Shear stress Distribution with r/r_d for a solid disc

We also observe from these results that the shear stress does not have excessive increase with the increase in the surface velocity. Dotted lines give the results obtained from Eq.(55) for the case of solid disc. For the annular disc of radius ratio $r_i/r_d = 1/5$ and v = 0.3, shear stress distribution is plotted in Figure 7. It is also observed from this figure that the shear stress does not seemingly depend on the surface velocity. Dotted lines give the values obtained by means of Eq. (55).

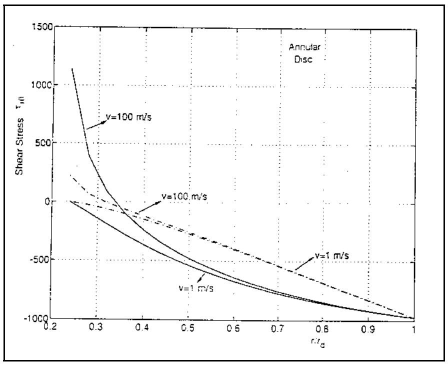

Figure 7. Shear stress distribution with r/r_d for an annular disc

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