

# EXPERIMENTAL INVESTIGATION OF THE PHENOMENON OF BUCKLING FOR STEEL AND ALUMINIUM STRUTS

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## ABSTRACT

The experiment was carried out to investigate the phenomenon of buckling using simple struts. These results were then compared with the theoretical predictions. Three steel struts of different length were used in the experiment; one of them had fix/pinned-end all the others had pin/pin-end joint. The applied load was placed at different eccentricities for each strut. Six aluminium pin-end struts of varying length were also tested. The measured critical load for each strut was compared against the corresponding Euler and Southwell predictions. For a steel strut, it would be expected that buckling would be symmetrical for left and right eccentricities. However, this was not the case due to imperfections in the struts. The struts buckled with half sine-wave and if one end of the strut was fixed the effective length was reduced and the critical load was increased. In the case of the aluminium struts, due to plastic behaviour in the deformation it was much harder to find the critical load. For steel struts both Euler and Southwell predictions were close to initial estimates of critical load.

**Keywords:** Experimental Investigation of Buckling, Steel and Aluminium Struts, Euler and Southwell Predictions

## ÇELİK VE ALÜMİNYUM ÇUBUKLARDA BURKULMANIN DENEYSEL İNCELENMESİ

### ÖZET

Bu çalışmada burkulma olayı basit çubuklar kullanılarak denendi. Elde edilen değerler daha sonra teorik tahminlerle karşılaştırıldı. Deney sırasında değişik uzunluklarda üç çelik çubuk kullanıldı. Bunlardan birisi sabit olarak diğer ikisinde hareket edebilecek şekilde mesnetlendi. Yükleme her bir çubuk için farklı olmak üzere eksantrik olarak sağlı sollu yüklemeye tabii tutuldu. Buna ek olarak değişik uzunluklarda altı alüminyum çubuk üzerinde deney yapıldı. Her bir çubuk için ölçülen kritik yüklemeye buna karşılık gelen Euler ve Southwell tahminleriyle karşılaştırıldı. Çelikten üretilmiş bir çubuk için; burkulmanın, simetri eksenin sağında ve solunda aynı olması beklenirken çubukların yapısındaki kusurlar nedeniyle aynı olmamaktadır. Yapılan deneylerde çubukların yarım sinus dalgası şeklinde burkulduğu; çubuğun bir tarafının sabit mesnetli olarak yüklendiği durumda etkin uzunluğun azaldığı ve kritik yükün arttığı gözlenmiştir. Alüminyum çubuklarda deformasyon sırasındaki plastik davranış nedeniyle kritik yükün tespiti çok büyük zorluk göstermektedir. Çeliklerden üretilmiş çubuklar için hem Euler hem de Southwell formülleri kritik yükün tahmininde çok yakın sonuçlar vermektedir.

**Anahtar Kelimeler:** Burkulmanın Deneysel İncelenmesi, Çelik ve Alüminyum Çubuk, Euler ve Southwell Tahminleri

## 1. INTRODUCTION

Buckling is a mode of deformation which develops in a direction or plane normal to that of the loading which produces it. Therefore deformation changes rapidly with the change in the magnitude of applied loading. It

occurs in members and elements that are in a state of compression.

The simple test of buckling was analysed using struts which were initially straight and struts with eccentricities. Struts were compressed by equal and opposite axial forces.

The member's buckling resistance will increase with the bending stiffness of the member, and hence with the thickness of the depth of its section measured in the plane of buckling deformation; also it decreases as the member length is increased. Thus buckling resistance is low if a member is slender and high if it is stocky.

Buckling is of particular interest with steel members because they tend to be of slender form compared, for example, with eccentricity members. However, it is not only slenderness of a member as a whole that leads to buckling. The thin elements of spring steel plate or sheet may have individually experience localised buckling effect when subjected to compressive stress.

This experimental work was carried out to investigate buckling in the context of compressed struts, and identifies the main parameters that govern buckling behaviour. Firstly we considered the elastic behaviour of an idealised strut having perfect geometry, such as no initial out-of straightness or eccentricity of loading using spring steel. Then we examined the effect of assuming either ideal rigid plastic or ideal elastic-plastic material behaviour in the absence of residual stress. Finally we were studied in turn the influences of imperfect geometry, residual stresses and more general elastic-plastic material behaviour.

## 2. THEORY

The experiment was carried out to see if Euler's prediction could be relied upon in practice. When the applied load reaches the critical load elastic buckling occurs. Euler prediction for pin-end strut is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (1.a)$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2} \quad (1.b)$$

where  $\sigma_{cr}$  is critical stress (N/mm<sup>2</sup>), A is cross section area (mm<sup>2</sup>),  $P_{cr}$  is critical load (N), E is elastic modulus (N/mm<sup>2</sup>), I is second moment of area (mm<sup>4</sup>) and L is specimen length (mm). The formulation for fix-end strut is given similarly by

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{(0.7)AL^2} \quad (2)$$

$P_{cr}$  can be calculated by using Southwell method, i.e.

$$\frac{1}{P_{cr}} = \frac{\delta/P}{\delta} = \frac{dy}{dx} \quad (3)$$

$$\delta = -a_0 + P_{cr} (\delta/P)$$

where  $a_0$  is initial imperfection of the strut. If r is the radius of gyration of the cross section then the second moment of area is given by

$$I = Ar^2 \quad (4)$$

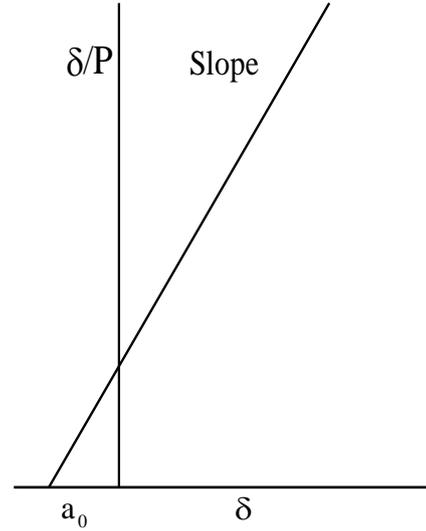


Figure 1 Southwell Plot

then

$$\sigma_{cr} = \frac{\pi^2 E r^2}{L^2} = \frac{\pi^2 E}{\lambda^2} \quad (5)$$

where  $\lambda$  is the slenderness ratio defined by

$$\lambda = \frac{L}{r} \quad (6)$$

then  $\sigma_{cr}$  can be written by

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2} \quad (7)$$

where  $r^2 = (I/A)$ ,  $r \approx (t/3.465)$  and t is thickness of strut the strut (mm). If  $(L/r)^2 > (\pi^2 E / \sigma_y)$  then the strut is slender and  $(L/r)^2 < (\pi^2 E / \sigma_y)$  is stocky. Critical slenderness ratio is given by

$$\lambda_T = \pi \sqrt{(E / \sigma_y)} \quad (8)$$

where  $\sigma_y$  is material yield stress. For struts having  $\lambda < \lambda_T$  failure is by plastic squashing compressive

failure while for struts having  $\lambda > \lambda_T$  failure is by elastic buckling.  $\lambda_T$  calculated for aluminium was 79.86.

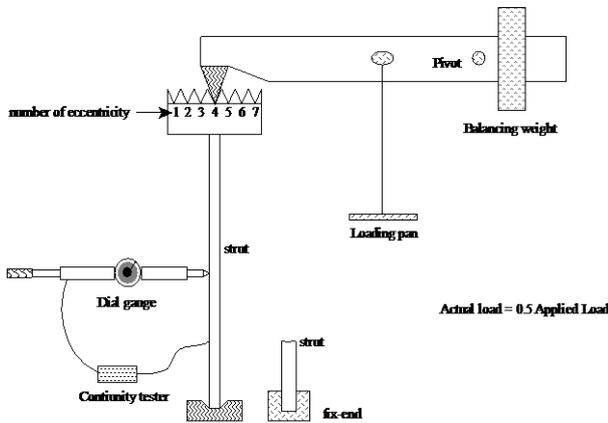


Figure 2. Buckling apparatus.

### 3. PROCEDURE

The apparatus was set up as shown in Figure 1 which was used for measuring strut buckling. Two types of material were used: three steel strut of different length with pin and fixed ends, and six aluminium pin-end struts of different lengths. Each strut was placed in the buckling apparatus with the top loaded at eccentricities varying between  $\pm 12$  and 0 mm. A dial gauge reading of deflection was obtained for each load, and then plotted on a graph versus critical load (Figure 3,5,7).

The critical load was estimated by assuming the behaviour of a perfect strut to be an asymptote to the curves produced with various eccentricities. A Southwell plot of deflection/load versus deflection was drawn Figure 4,6,8. The critical load was calculated and compared with the Euler equation (Eq. 1.a).

The steel strut was removed and six aluminium pin-end struts were used. Also critical values were calculated for aluminium. These values can be seen on table 1.

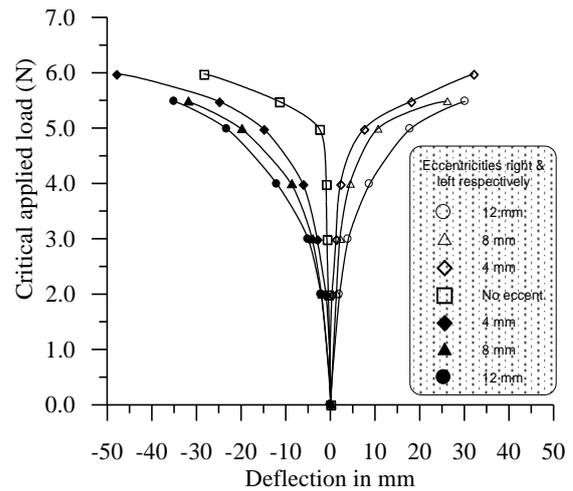


Figure 3. Load against deflection for a 430 mm long steel strut (Thickness=0.66 mm & width=25.2 mm pin-end)

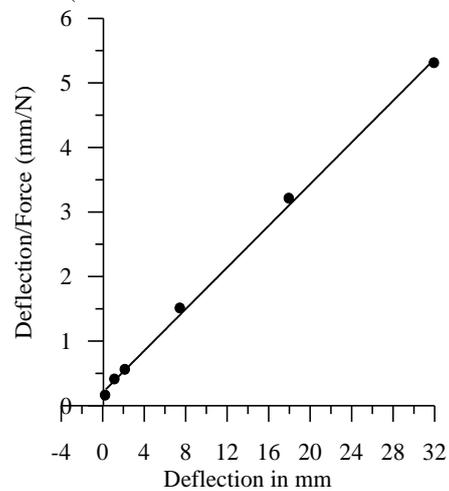


Figure 4. Southwell plot for a 430 mm long steel strut (Thickness=0.66 mm & width=25 mm pin-end strut)

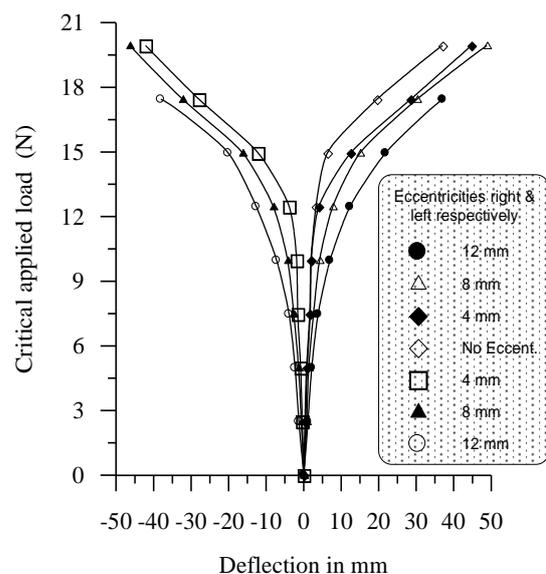


Figure 5. Load against deflection for a 252 mm long steel strut (Thickness=0.66 mm & width=25.2 mm pin-end)

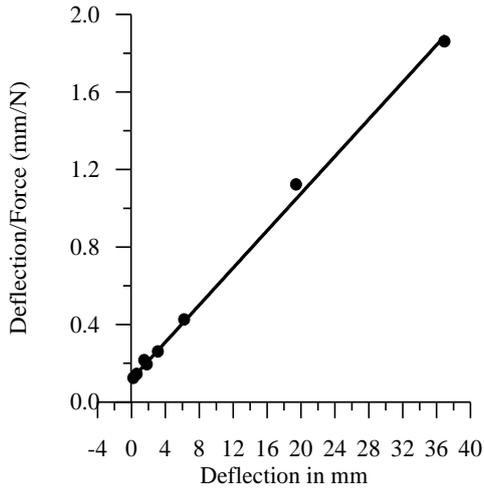


Figure 6. Southwell plot for a 252 mm long steel strut (Thickness=0.66 mm & width=25.2 mm pin-end strut)

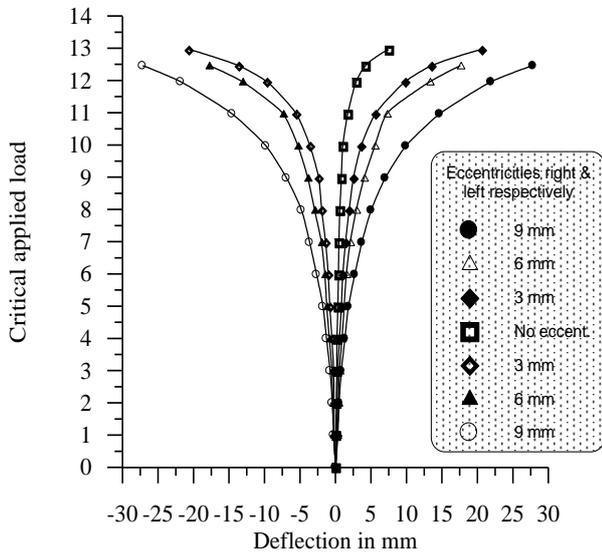


Figure 7. Load against deflection for a 425 mm long steel strut (Thickness=0.66 mm & width=25 mm fix-end)

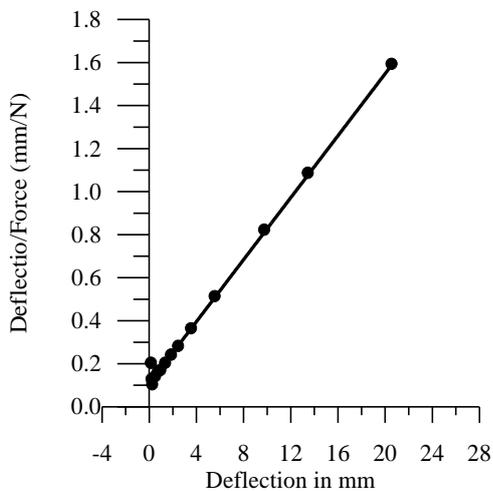


Figure 8. Southwell plot for a 425 mm long steel strut (Thickness=0.66 mm & width=25 mm fix-end strut)

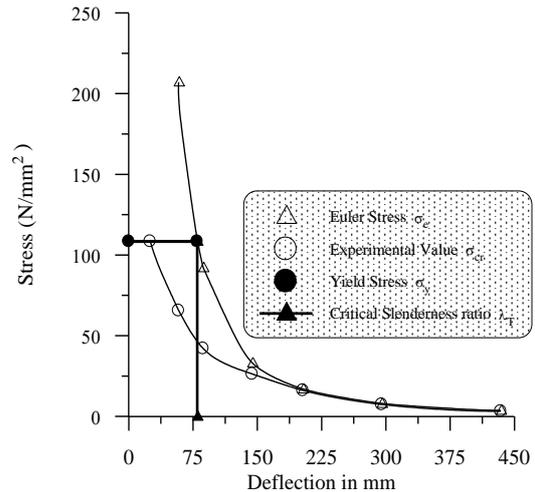


Figure 9 Limiting stress for a pin-end aluminium strut based on the stress-strain curve

Two graphs were drawn for aluminium struts (Figure 9). The yield stress was determined experimentally and plotted in Figure 9.

All the results were compared to find if Southwell and Euler were useful in practical situations

#### 4. RESULTS

Results of  $P_{cr}$  for all struts were taken from graphs and calculated from Euler and Southwell predictions. The result are presented in Table 2

Table 2 Experimental and theoretical calculation of critical load

Specimen No	1	2	3	4	5	6
Thick. t(mm)	1.2	1.2	1.2	1.2	1.2	1.2
Width w(mm)	25.2	25.2	25.2	25.2	25.2	25.2
Length L(mm)	150	102	70	50	30	20
Slenderness Ratio $\lambda = L / r$ $r=t/3.464$	434	295	203	144	86.6	58
Experimental Yield stress $\sigma_y = P_y / A$ MPa	108.3	108.3	108.3	108.3	108.3	108.3
Buckling Load Per (N)	94	220	480	792	1270	1980

#### 5. DISCUSSION AND CONCLUSION

It is concluded that all the struts buckled in a half sine-wave. When one end was fixed the effective length of strut was reduced to value of 0.7L. The critical load therefore increased. It can be seen from graphs that the struts should buckle symmetrically when eccentric load to the right and left are applied but initial imperfections in the struts did not allow this to happen. For the 430 mm strut the graph gave us a critical load of 6.22 N Southwell gave us 6.29 N and Euler 6.6 N. The

difference between Euler experiment and Southwell experiment were 6% and 1%. For steel struts the experimental error varied between 1% and 6%. This error could be due to the apparatus and due to procedural errors. From these results, both Southwell and Euler predictions can be used in practical situations. Since the results were very close to the observed cases.

The critical buckling load for a pin-end strut Figure 3, is compared to  $P_{cr}$  for a fix-end strut, Figure 7. Fix-end strut buckled at twice the axial force compared with the pin-end one because of the fix-end. This is due to the fact that the joint is prevented from moving freely when the strut was loaded by axial load.

Effect of length on the critical compressive load for pin-end aluminium struts were calculated and were then recorded on a table 1. These results were drawn

Table 1. Effect of length on the critical compressive load for pin-end aluminium struts

End type	pin/pin-end				fix/pinned-end	
Dimension of steel strut	430		252		425	
$P_{cr}$ (N) Theoretical	Euler	Southwell	Euler	Southwell	Euler	Southwell
	6.6	6.29	19.27	19.38	13.82	13.75
$P_{cr}$ (N) Experiment	From figure 3		From figure 5		From figure 7	
	6.22		19.93		13.45	
Difference %	Euler	Southwell	Euler	Southwell	Euler	Southwell
	6	1.1	3.3	2.7	2.7	2.23

on graph (Figure 9). It can be seen from figure 2 when the length of the specimen is too small and  $\lambda < \lambda_T$  then plastic squashing occurs when the length of the specimen is too long and  $\lambda > \lambda_T$  failure is called elastic buckling.

Consequently the designer must always avoid elastic or plastic buckling. Plastic squashing will be a stable mode of failure predicted by knowledge of the yield stress but the value of  $\lambda_T$  has been shown to need reducing by a factor of  $(30/79.86=0.37)$  as shown figure 9.

## 6. REFERENCES

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