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Determination of bounding frequencies of cylindrical shells using a periodic structure wave approach with Rayleigh-Ritz method

Rayleigh-Ritz metodu ile periyodik yapı dalga yaklaşımı kullanılarak silindirik kabukların sınır frekanslarının belirlenmesi

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Abstract

In this paper, periodic structure theory with the wave approximation is used to present a simple approximate solution technique to characterize wave motions propagating in periodic line-supported cylinders in the circumferential direction. To develop displacement functions that adhere to Floquet's concept, a combination of simple beam functions of the bounding modes(BM)of propagation bands(PB) of a periodic beam are formulated. This study is developed for the motion type known as a plane wave. Consequently, only waves that are simply propagating without attenuation are taken into account. The circumferential modes of a single periodic curved panel (unit cell) have been defined in terms of classical beam functions that satisfy Floquet's wave principle, but the axial modes are thought to be sinusoidal waves. Displacement functions are used to strain energy and kinetic energy expressions. The Rayleigh-Ritz technique is then used to generate the stiffness and mass matrices of the periodic unit cell. By solving the eigenvalue equation, phase-frequency relation is obtained. It has also been possible to predict the bounding frequencies of the PB for various axial modes of a cylindrical shell with a certain circumferential phase constant. The findings are then put through comparison with those outlined in the literature. Further, the bounding frequency results for the optimum periodic curved panel which gives lowest frequency for a given cylindrical shell geometry are also found out. It has been found that the current beam function with a periodic structure (PS) wave approach can find the bounding frequencies (BF) and bounding modes (BM) with reasonable accuracy.

Keywords: Cylindrical shell, Curved panel, Wave propagation, Bounding frequency, Beam function, Rayleigh-Ritz method

Öz

Bu çalışmada, dalga yaklaşımı ile periyodik yapı teorisi, çevresel yönde periyodik çizgi destekli silindirlerde yayılan dalga hareketlerini karakterize etmek için basit bir yaklaşım çözüm tekniği sunmak için kullanılmaktadır. Floquet'nin kavramına uygun yer değiştirme fonksiyonları geliştirmek için, periyodik bir kirişin yayılma bantlarının (PB) sınırlar modlarının (BM) basit kiriş fonksiyonlarının bir kombinasyonu formüle edilmiştir. Bu çalışma düzlem dalga olarak bilinen hareket türü için geliştirilmiştir. Sonuç olarak, yalnızca zayıflama olmaksızın yayılan dalgalar dikkate alınmıştır. Tek bir periyodik eğri panelin (birim hücre) çepeçevrgi modları, Floquet'in dalga prensibini karşılayan klasik ışın fonksiyonları açısından tanımlanmıştır,ancak eksenel modların sinüzoidal dalgalar olduğu düşünülmektedir.Yer değiştirme fonksiyonları,gerinim enerjisi ve kinetik enerji ifadelerini germek için kullanılır. Rayleigh-Ritz tekniği daha sonra periyodik birim hücrenin sertlik ve kütle matrislerini oluşturmak için kullanılır. Özdeğer denkleminin çözülmesiyle faz-frekans ilişkisi elde edilir. Belirli bir çepeçevrgi faz sabiti ile silindirik bir kabuğun çeşitli eksenel modları için PB'nin sınırlar frekanslarını tahmin etmek de mümkün olmuştur. Elde edilen bulgular daha sonra literatürde belirtilenlerle karşılaştırılmıştır. Ayrıca, belirli bir silindirik kabuk geometrisi için en düşük frekansı veren optimum periyodik kavisli panel için sınırlar frekansı sonuçları da bulunmuştur. Periyodik yapı (PS) dalga yaklaşımına sahip mevcut ışın fonksiyonunun sınırlar frekansları (BF) ve sınırlar modları (BM) makul bir doğrulukla bulabildiği tespit edilmiştir.

Anahtar kelimeler : Silindirik kabuk, Kavisli panel, Dalga yayılımı, Sınırlama frekansı, Işın işlevi, Rayleigh-Ritz yöntemi

1 Introduction

The use of the wave propagation method to the dynamics of periodic structures has proven to be an effective tool. Some alike periodic elements connected end-to-end and/or connected side-by-side to form a complete structure are the basic constituents of a periodic structure(PS). Engineering structures such as high-rise buildings[1], elastic foundations[2], elevated guideways, multiple-span bridges, train tracks, multiple-bladed turbines, wings and fuselages of aeroplane, gas pipelines, and reinforced shells/plates in the marine and aerospace industries have been or are treated as periodic.

These structures all can transmit waves in discrete frequency bands known as "propagation bands(PB)" or "pass bands," while preventing waves in other frequency bands from propagating, known as "attenuation bands". A thorough review of the works of literature concerning dynamic assessments of PS is provided by Mead[3].

Solid-state physicists were the first to apply the wave propagation approach to research the dynamics of periodic unit cells [4]. The approach was broadened to take into account the study of flexural waves that occur on periodic beam and plate structures in engineering [5]-[9] meeting Floquet's criterion. The phase constant change with the frequency of infinitely long uniform beams and plates on equally spaced rigid supports has been estimated to represent the dispersion relationship of flexural waves. By discretizing the PB's(dispersion curve), it is possible to determine the eigenfrequencies of finite arrays of structures [10]. Based on references[6]-[7], there exist alternating bands of propagation of waves and decay for a

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continuously periodic supporting beam. Mead and Parthan [9] successfully determined the PB by using beam functions and polynomial functions approximations and analyzing the same problem[6] using the periodic structure (PS) technique. Such boundary mode products are utilized in [9] to assess the multiple supported periodic flat panel's dispersion surfaces when exact results are not accessible.

It is requisite to understand the vibration attributes of stiffened/ line supported cylindrical shells and panels to analyze acoustic response in engineering structures.. In this article an analytical method is proposed to determine the bounding frequency of such structures by considering them as an arrangement of a number of identical cylindrical curved panels using the periodic structure wave approach. Mead and Bardell [11]-[12] studied vibration analysis(free) of cylinders with distinct stiffeners in the axial direction and circumferential direction using shells differential equations and PS theory. In comparison with other techniques, this method begins by taking the structure's single-bay(unit cells) dispersion curve into account before calculating the natural frequencies of the whole system. Shell vibration problem has also been studied using the PS concept combined with hierarchical FEM [13]-[14] for obtaining dynamic characteristics (Propagation constants/surfaces versus frequency) of orthogonally simple line supports (LS) and stiffened cylinders. Accorsi and Bennett have applied FEM to determine dispersion curves/surfaces in orthogonal stiffened cylinders. Identical axially and circumferentially spaced stiffeners were assumed, and complex, real and imaginary propagations constants are determined for a single periodic unit [15]. Laurent et al. [16] propose a semi-analytical technique for modeling the vibroacoustic of immersed cylinder strengthened by periodic axisymmetric frames. It is calculated how Floquet's harmonics and support position affects acoustic emission. The basic procedure to compute the free wave propagation in a 1-D(one dimensional) or quasi-1D periodic continuous system, uniform cylindrical shells, and flat panels have been employed. The PS theory with FEM has been used to study free wave propagation and generate dispersion relations in periodic flat panels[17], unsupported cylindrical shells [18]-[19] and circular ring[20], axial periodic LS (line supports) infinitely long curved panels[21], and orthogonally periodic LS curved panels[22]. Whether a structure is an open structure or a closed structure, the dispersion curve remains the same.

Each periodic element is a segment of the shell between two successive nodal points in the case of a cylindrical shell. The most obvious choice in the periodic structure analysis of shell structure is the optimum periodic curved panel, which is proposed [19],[24]. This optimum (ideal) periodic angle corresponds to the lowest frequency of the curved panel (optimum) dimension vibrating in the first axial and first circumferential modes. Additionally, this will be the whole circular cylindrical shell's lowest frequency. Flutter analysis of isolated flat and curved panels is presented using high precision efficient arbitrary triangular finite element method for different constraint conditions on its edges [25]. The PS wave technique has been used to study the 1-D axial wave motions in a long periodically supported cylindrical curved panel exposed to supersonic airflow along its generator [26]. All of a finite structure's dynamic attributes can be determined from a single phase-frequency curve or surface owing to periodic structure analysis. Free wave propagation computing has been adapted to cylindrical shells with periodic reinforced stiffeners/line supports (LS) along the circumference or length [11]-[14]. It has been adapted for an unstiffened shell [18]-[19] and the results have been correlated with those obtained by classical Warburton's approach [27]. Even though these phenomena are widely understood, the majority of literature papers on periodic engineering structures focus on the development of theoretical and computational methodologies to understand their wave propagation behavior and features.

Therefore, in this paper,a new formulation has been presented using the periodic structure theory wave approach(plane wave motion type) with the Rayleigh-Ritz method to obtain the bounding natural frequency of a thin cylindrical structure with periodic line supports (LS) along the circumference as depicted in Figure 1a. Here, classical beam functions[9],[28] satisfying the Floquet's wave boundary periodicity conditions[3]-[4] have been used for circumferential modes of a periodic unit cell i.e. curved panel as shown in Figure 1b, while the axial modes are assumed as sinusoidal waves. The natural vibration frequencies of a complete cylindrical shell/curved panel have been obtained by the wave method using the PS concept. The BF (bounding frequencies) are found for different axial modes of propagation bands(PB) or phase-frequency curves. The findings are compared with the literature data[13]. Further, the bounding frequency results for the optimum periodic curved panel which gives lowest frequency for a given cylindrical shell geometry are found out. It is found that the current beam function with wave approach able to find the bounding frequencies(BF) and bounding modes(BM) with reasonable accuracy. The advantage of this method is matrices of small order need to be evaluated. This research can be used to improve the design and analysis of cylindrical shells that are better resistant to vibration and has the potential to be applied to other engineering problems related to wave propagation in periodic structures to predict the stop band (bounding frequency(BF) of propagation band or phase-frequency curve) based on the unit cell modeling.

2 Mathematical formulation

The analysis of (PS) periodic structures (such beams, plates, or shells) introduces a parameter (δ) termed as "propagation constant" by using the "Floquet principle," which connects the vectors at two corresponding sets 'a' and 'b' in adjacent repeating elements:

$$\text{Vector at 'b'} = (\text{Vectorat 'a'}) * \exp(\delta) \quad (1)$$

where the propagation constant, $\delta = \delta_r + i\delta_i$ ($i = \sqrt{-1}$) is the basic complex form. The phase lag or lead of the vibration at point 'b' concerning point 'a' (Figure 1 a) is represented by the portion δ_i (ϵ) that is imaginary, whilst the real part (δ_r) denotes the spatial increment or decrement in amplitude.

In the current work using the periodic structure theory in a wave approach and beam functions that satisfy the wave boundary requirements following Floquet's principle, the natural frequencies have been predicted for a cylindrical shell with circumferential periodic LS as shown in Figure 1a.

(a)

(b)

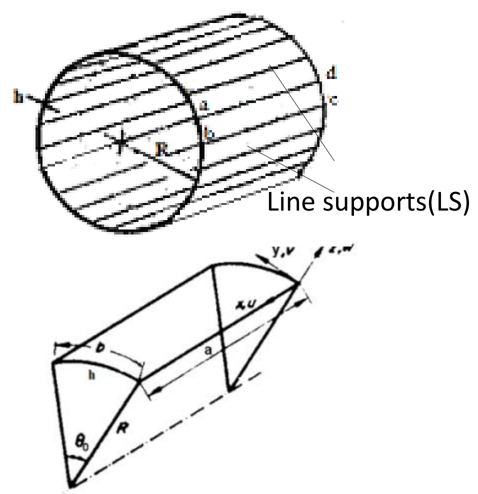


Figure 1. (a) Cylindrical shell showing periodic unit (a-b-c-d)
(b) Curved panel (one periodic unit cell)

Displacement functions that fulfil Floquet's principle are formed by combining simple beam functions of a periodic multi-supported beam's BM of PB. The LS and the circular edge supports are assumed as simple, and radially non-deflecting. For this study, the beam functions (f and g) [9] that meet wave boundary requirements are used as follows:

$$f(\eta,\varepsilon)=\sin\left(\frac{\epsilon}{2}\right)\sin(\pi\eta)-\left(\frac{i}{2}\right)\cos\left(\frac{\epsilon}{2}\right)\sin(2\pi\eta)$$

$$g(\eta,\varepsilon)=\cos\left(\frac{\epsilon}{2}\right)\phi_s(\eta)+i(\gamma^2)\sin\left(\frac{\epsilon}{2}\right)\phi_{s+1}(\eta) \qquad (2a)$$

$$\gamma^2=\left(\frac{\lambda_s}{\lambda_{s+1}}\right)^2;\ \epsilon=\epsilon_y$$

The modes chosen are (i) the bound modes(sinusoidal modes) for axial wave motion, and (ii) the bound modes (beam function modes that satisfy Floquet's wave principle for the wave motion along the circumference, viz:

$$U(\xi,\eta,t)=u(\xi,\eta)e^{i\omega t};u=(u_1f+u_2g)\cos(r\pi\xi)$$

$$V(\xi,\eta,t)=v(\xi,\eta)e^{i\omega t};v=\left(v_1f_{,\eta}+v_2g_{,\eta}\right)\sin(r\pi\xi) \qquad (2b)$$

$$W(\xi,\eta,t)=w(\xi,\eta)e^{i\omega t};w=(w_1f+w_2g)\sin(r\pi\xi)$$

where s is an odd positive integer. ϕ_s is the fixed-end beam function of the nth mode[28],[21].

η=y/b and ξ=x/a. η and ξ vary from 0 to 1.`$f_{,\eta}$ and $g_{,\eta}$ are the first differential of the function f and g with respect to η.

The above displacement functions meet Floquet's principle (Eq. (1)) for the periodic (repeating) element in Figure 1b of the cylindrical shell in Figure 1a as follows:

i) Along the generator of periodic cylindrical panel

$$u_{,\xi}(0,\eta)=u_{,\xi}(1,\eta)=0;$$
$$v(0,\eta)=v(1,\eta)=0;$$
$$w(0,\eta)=w(1,\eta)=0;$$
$$w_{,\xi\xi}(0,\eta)=w_{,\xi\xi}(1,\eta)=0; \qquad (3)$$

$$\text{Where, } u_{,\xi}=\frac{\partial u}{\partial\xi};w_{,\xi\xi}=\frac{\partial^2 w}{\partial\xi^2}$$

ii) Along LS for the flexural wave's for circumferential propagation

$$u(\xi,1)=u(\xi,0)=0;\ u_{,\eta}(\xi,1)=u_{,\eta}(\xi,0)e^{-i\epsilon};$$
$$u_{,\eta\eta}(\xi,1)=u_{,\eta\eta}(\xi,0)e^{-i\epsilon};$$
$$v(\xi,1)=v(\xi,0)=0;\ v_{,\eta}(\xi,1)=v_{,\eta}(\xi,0)e^{-i\epsilon};$$
$$w(\xi,1)=w(\xi,0)=0;\quad w_{,\eta}(\xi,1)=w_{,\eta}(\xi,0)e^{-i\epsilon};$$
$$w_{,\eta\eta}(\xi,1)=w_{,\eta\eta}(\xi,0)e^{-i\epsilon} \qquad (4)$$

$$\text{Where, } u_{,\eta}=\frac{\partial u}{\partial\eta},\ u_{,\eta\eta}=\frac{\partial^2 u}{\partial\eta^2},v_{,\eta}=\frac{\partial v}{\partial\eta},\ v_{,\eta\eta}=\frac{\partial^2 v}{\partial\eta^2},$$
$$w_{,\eta}=\frac{\partial w}{\partial\eta},\quad w_{,\eta\eta}=\frac{\partial^2 w}{\partial\eta^2}$$

The component phase difference between neighbouring LS is represented by the phase constant ($\epsilon=\delta_i=\epsilon_y$), where x, y are the coordinates of a point with the left support as the origin.

Rayleigh quotient for a single beam's frequency employing approximately complex mode waveforms with a known imaginary part (i.e. phase parameter or constant) ϵ_y [3] of the propagation constant (δ) for a plane wave:

$$\omega^2=\frac{\int_0^L EI\left|\frac{d^2w}{dx^2}\right|^2dx}{\int_0^L\rho A|w|^2\,dx} \qquad (5)$$

The integrals' modulus signs cause this to be different from the typical Rayleigh quotient. The cylindrical panels' changed strain and kinetic energy expressions are shown below:

The strain energy (U) of the periodic unit cell/repeating element in Figure 1b is:

$$U=C\int_0^1\int_0^1\left[\begin{matrix}(D_1^2(|u_{,\xi}|)^2)+(D_2^2(|v_{,\eta}|)^2)+\left(D_2(|w||v_{,\eta}|^*+|v_{,\eta}||w|^*)\right)+(|w|)^2+(\nu D_1D_2(|u_{,\xi}||v_{,\eta}|^*+|v_{,\eta}||u_{,\xi}|^*))\\+\left(\nu D_1(|w||u_{,\xi}|^*+|u_{,\xi}||w|^*)\right)+\left(0.5\nu'D_1^2(|v_{,\xi}|)^2\right)+\left(0.5\nu'D_1D_2(|v_{,\xi}||u_{,\eta}|^*+|u_{,\eta}||v_{,\xi}|^*)\right)\\+\left(0.5\nu'D_2^2(|u_{,\eta}|)^2\right)+\left(\beta D_1^4(|w_{,\xi\xi}|)^2\right)+\left(\beta D_2^4(|w_{,\eta\eta}|)^2\right)-\left(\beta D_2^3(|w_{,\eta\eta}||v_{,\eta}|^*+|v_{,\eta}||w_{,\eta\eta}|^*)\right)\\+\left(\beta\nu'D_2^2(|v_{,\eta}|)^2\right)+\left(\beta\nu D_1^2D_2^2(|w_{,\xi\xi}||w_{,\eta\eta}|^*+|w_{,\eta\eta}||w_{,\xi\xi}|^*)\right)-\left(\beta\nu D_1^2D_2(|w_{,\xi\xi}||v_{,\eta}|^*+|v_{,\eta}||w_{,\xi\xi}|^*)\right)\\+(2\nu'D_1D_2(|w_{,\xi\eta}|)^2-\left(2\beta\nu'D_1^2D_2(|w_{,\xi\eta}||v_{,\xi}|^*+|v_{,\xi}||w_{,\xi\eta}|^*)\right)+(2\beta\nu'D_1^2(|v_{,\xi}|)^2)\end{matrix}\right]d\xi d\eta \qquad (6)$$

$$\text{Where, } C=\frac{Ehab}{2R^2(1-\nu^2)}\ ;\ D_1=\frac{R}{a}\ ;\ D_2=\frac{R}{b}\ ;(1-\nu)=\nu'$$

A periodic unit cell /repeating element's kinetic energy is:

$$T=\frac{\rho hab\omega^2}{2}\int_0^1\int_0^1[|u|^2+|v|^2+|w|^2]d\xi d\eta \qquad (7)$$

The formulas for strain energy and kinetic energy make use of the displacement functions (Eq. (2)). The periodic unit cell stiffness and mass matrices are then derived using the Rayleigh-Ritz technique.

Rayleigh quotient is $\Omega^2=U/T^*$

$$T^*=\frac{\rho hab}{2}\int_0^1\int_0^1[|u|^2+|v|^2+|w|^2]d\xi d\eta \qquad (8)$$

Using the Rayleigh-Ritz method,

$\frac{\partial \Omega^2}{\partial q_i^*} = 0$; $(q_i = u_1, u_2, v_1, v_2, w_1, w_2)$,one eventually obtains

$$([K] - \Omega^2[M])\{q\} = \{0\} \quad (9)$$

For non-trivial solutions (if and only if the determinant of a matrix is zero), the linear algebraic equations of motion are found for the coefficients q_i (i=1.....6) as follows:

$$\det \mid [K] - \Omega^2[M] \mid = \{0\} \quad (10)$$

Ω is dimensionless frequency. We now have (6 x 6) stiffness and (6 x 6) mass matrices. These are as follows.

2.1 Stiffness matrix

$$[K] = 0.5C[K_{ij}] \quad (11)$$

Where, K_{ij} are

$$K_{11} = (r^2\pi^2 D_1^2 I_1 + 0.5\nu' D_2^2 I_4);$$

$$K_{12} = (r^2\pi^2 D_1^2 I_2 + 0.5\nu' D_2^2 I_5)\ ;$$

$$K_{13} = r\pi(\nu D_1 D_2 I_7 + 0.5\nu' D_1 D_2 I_4);$$

$$K_{14} = r\pi(\nu D_1 D_2 I_8 + 0.5\nu' D_1 D_2 I_5);$$

$$K_{15} = r\pi(\nu D_1 I_1);\ K_{16} = r\pi(\nu D_1 I_2);$$

$$K_{22} = (r^2\pi^2 D_1^2 I_3 + 0.5\nu' D_2^2 I_6);$$

$$K_{23} = r\pi(\nu D_1 D_2 I_8 + 0.5\nu' D_1 D_2 I_5);$$

$$K_{24} = r\pi(\nu D_1 D_2 I_9 + 0.5\nu' D_1 D_2 I_6);$$

$$K_{25} = r\pi(\nu D_1 I_2);\ K_{26} = r\pi(\nu D_1 I_3);$$

$$K_{33} = r^2\pi^2[0.5(1 + \beta\nu')D_2^2 I_{10} + \nu'(0.5 + 2\beta)D_1^2 I_4];$$

$$K_{34} = r^2\pi^2[0.5(1 + \beta\nu')D_2^2 I_{11} + \nu'(0.5 + 2\beta)D_1^2 I_5]\ ;$$

$$K_{35} = D_2[(1 - \beta r^2\pi^2 D_1^2)I_7 - \beta D_2^2 I_{10} - \beta\nu' r^2\pi^2 D_1^2 I_4];$$

$$K_{36} = D_2[(1 - \beta r^2\pi^2 D_1^2)I_8 - \beta D_2^2 I_{11} - \beta\nu' r^2\pi^2 D_1^2 I_5];$$

$$K_{44} = [(1 + \beta\nu')D_2^2 I_{12} + \nu'(0.5 + 2\beta)r^2\pi^2 D_1^2 I_6];$$

$$K_{45} = D_2[(1 - \beta r^2\pi^2 D_1^2)I_7 - \beta D_2^2 I_{10} - \beta\nu' r^2\pi^2 D_1^2 I_4]\ ;$$

$$K_{46} = D_2[(1 - \beta r^2\pi^2 D_1^2)I_9 - \beta D_2^2 I_{11} - \beta\nu' r^2\pi^2 D_1^2 I_6];$$

$$K_{55} = [(1 + \beta r^4\pi^4 D_1^4)I_1 + \beta D_2^4 I_{10} + 2\beta\nu' r^2\pi^2 D_1^2 D_2^2 I_4 + \beta\nu r^2\pi^2 D_1^2 D_2^2 I_7];$$

$$K_{56} = [(1 + \beta r^4\pi^4 D_1^4)I_2 + \beta D_2^4 I_{11} + 2\beta\nu' r^2\pi^2 D_1^2 D_2^2 I_5 + \beta\nu r^2\pi^2 D_1^2 D_2^2 I_8];$$

$$K_{66} = [(1 + \beta r^4\pi^4 D_1^4)I_3 + \beta D_2^4 I_{12} + 2\beta\nu' r^2\pi^2 D_1^2 D_2^2 I_6 + \beta\nu r^2\pi^2 D_1^2 D_2^2 I_9];$$

The integration I_i are

$$I_1 = \int_0^1 f f^*\, d\eta;\ I_2 = \int_0^1 f g^*\, d\eta;\ I_3 = \int_0^1 g f^*\, d\eta;$$

$$I_4 = \int_0^1 f_{,\eta} f_{,\eta}{}^*\, d\eta;\ I_5 = \int_0^1 f_{,\eta} g_{,\eta}{}^*\, d\eta = \int_0^1 g_{,\eta} f_{,\eta}{}^*\, d\eta;$$

$$I_6 = \int_0^1 g_{,\eta} g_{,\eta}{}^*\, d\eta;$$

$$I_7 = \int_0^1 f f_{,\eta\eta}^*\, d\eta = \int_0^1 f_{,\eta\eta} f^*\, d\eta;$$

$$I_8 = \int_0^1 f g_{,\eta\eta}^*\, d\eta = \int_0^1 g f_{,\eta\eta}^*\, d\eta;$$

$$I_9 = \int_0^1 g g_{,\eta\eta}^*\, d\eta = -I_6;$$

$$I_{10} = \int_0^1 f_{,\eta\eta} f_{,\eta\eta}^*\, d\eta;$$

$$I_{11} = \int_0^1 f_{,\eta\eta} g_{,\eta\eta}^*\, d\eta = \int_0^1 g_{,\eta\eta} f_{,\eta\eta}^*\, d\eta;$$

$$I_{12} = \int_0^1 g_{,\eta\eta} g_{,\eta\eta}^*\, d\eta;$$

Where, $f_{,\eta}$ and $g_{,\eta}$ are the first differential of the functions f and g with respect to η; $f_{,\eta\eta}$ and $g_{,\eta\eta}$ are the second differential of the functions f and g with respect to η. f^*, and g^* are complex conjugate of functions f and g respectively.

2.2 Mass matrix

$$[M_{ij}] = \begin{bmatrix} \overline{I_1} & 0 & 0 \\ 0 & \overline{I_2} & 0 \\ 0 & 0 & \overline{I_3} \end{bmatrix} \quad (12)$$

Where,

$$\overline{I_1} = \begin{bmatrix} I_1 & I_2 \\ I_2 & I_4 \end{bmatrix}; \overline{I_2} = \begin{bmatrix} I_4 & I_5 \\ I_5 & I_6 \end{bmatrix}; \overline{I_3} = \begin{bmatrix} I_1 & I_2 \\ I_2 & I_3 \end{bmatrix}.$$

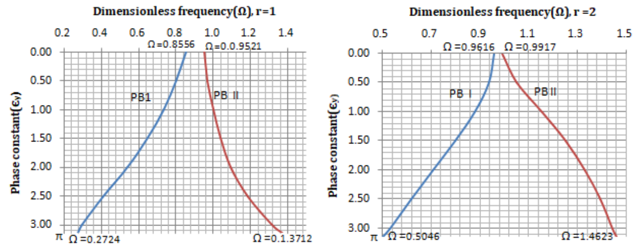
Equation (9) does not provide information about attenuated (or complex conjugate) waves. This is because ϵ_y is assumed to be purely imaginary. This situation is fulfilled when a "plane wave" propagates with a frequency(ω)across a line-supported cylinder along the y-axis. Each periodic unit oscillates in the same complex mode w(ξ,η)e $^{i\omega t}$ [3] but there will be a phase gap of ϵ_y between adjoining unit cells in the direction of the circumference (y or η).

The parameters used in the analysis are as follows. The geometrical parameters of curved panel(axial length(a), radius of curvature(R), circumferential length (b) and thickness(h)) and material parameter(Young's modulus of elasticity (E), density(ρ), Poisson's ratio (ν)) are supplied to analysis. The value of b(circumferential length of periodic curved panel or unit cell) will be used in the computation according to periodic angle(θ) of curved panel. Then, the value of phase constant ϵ_y can vary from 0 to π and the corresponding dimensionless frequency Ω is acquired from eigen value equation (9) for a given value of the axial mode(r). The dimensionless frequency value Ω at ϵ_y = 0 and π, are the bounding frequency of propagation band or phase-frequency curve.

3 Results and discussions

The geometric information and material attributes required to generate numerical results were taken from [11],[13]. The material is Aluminum and has the following properties: The modulus E is 70 GPa, the Poisson's ratio(ν) is 0.3, and the material density is 2700 kg/m^3. The axial length a = 0.135 m, the shell and panel radius R = 0.381 m, and the thickness h = 0.559 mm are the dimensional parameters of the curved panel and the full cylinder (Figure 1).

To compare the outcomes of the current formulation, eigenvalues of equation (9) were computed for a cylinder with randomly selected 44 equi-space circumferential simple (LS) line supports (periodic unit, i.e. curved panel with π/22 radian subtended angle at center)[13]. The dimensionless frequency Ω is determined using the present beam function with PS formulations for a given ϵ_y (0 to π) for different axial modes (r=1,2). The phase-frequency curves (propagation bands, PB I and PB II) are shown in Figure 3 for periodic angle π/22 radians. The BF compare well with the bounding frequencies of first two propagation bands (PB) obtained using the hierarchical function [13] and presented in Table 1.



(a) (b)

Figure 3. The first (PB I) and second(PB II) propagation bands (phase-frequency curve) for cylindrical shell with periodic angle θ = π/22 for axial mode (a) r =1, (b) r=2

The advantage of present method is that it requires evaluating matrices of small order. This method can envisage the dispersion relation in the "pure PB with no attenuations". This follows from the hypothetical presumption of the $\delta_r = 0$, which is generally not true.PB-associated attenuation was previously described by an exact analysis by Mead & Bardell [11].

The finite element code [17], [22]-[25] using high precision triangular finite elements of Cowper et al. [29] and analytical beam functions code[24] are used to find the frequencies (Ω) of the curved plate for various edge constraints. Simply-supported four edges: SSSS; clamped four edges: CCCC; simply supported straight sides clamped curved sides: SCSC; and simply supported curved sides straight sides clamped: CSCS; are various forms of edge constraints considered [24]. The results for the aforementioned frequencies for various axial (r) and circumferential (s) modes are well compared with the [13]. Now, these bounding frequencies (BF) obtained from the present formulation are compared with single curved panel free vibration frequencies with different edge boundary conditions and presented in Table 1.

Table 1. Comparison of BF (Ω) of phase-frequency curves or propagation bands(PB 1 and PB II) with periodic angle of π/22 radians for r=1, 2 with literature[13,24].

	Dimensionless bounding frequencies(Ω) and modes of propagation bands(PB) at phase constants ϵ_y=0 and ϵ_y=π			
	PB I		PB II	
r	$\epsilon_y = 0$	$\epsilon_y = \pi$	$\epsilon_y = 0$	$\epsilon_y = \pi$
1	0.8556⁺ 0.8556* 0.8548** s=2,(SSSS)	0.2724⁺ 0.2724* 0.2724** s=1,(SSSS)	0.9521⁺ 0.9517* 0.9521** s=1,(CSCS)	1.3712⁺ 1.3162* 1.3164** s=2,(CSCS)
2	0.9616⁺ 0.9636* 0.9661** s=2,(SSSS)	0.5046⁺ 0.5039* 0.5021** s=1,(SSSS)	0.9917⁺ 0.9887* 0.9921** s=1, (CSCS)	1.4623⁺ 1.4164* 1.4257** s=2,(CSCS)

(⁺Upper values are present PS approach; *Middle values are of[13]; and **lower values are dimensionless frequency and corresponding modes of single curved panel[24]; Lower values are modes (r, s) in the axial and circumferential direction of the single curved panel with SSSS or CSCS edges boundary conditions[24]).

There is a clear subtended angle in a curved panel in the earlier study [18],[19],[24] where the frequency is lowest. An optimum or ideal curved panel is one with this specific subtended angle (optimum angle). As a result, it becomes clear that choosing the repeating cells for shell analysis that would correspond to the lowest natural frequency with SSSS boundary conditions is reasonable. The lowest axial mode (r=1) of a circular, cylindrical shell as shown in Figure 1 with simply supported ends has the lowest radial vibration frequency, but the number of circumferential modes depends on the ratios of the shell's axial length (a) to its radius (R), as well as its thickness (h) to its radius (R) [27].

It was demonstrated in Figure 4a that the smallest Ω of value of 0.2516 corresponds to r=1 and N (circumferential full waves)=18 for radial vibration [18],[19],[24] using Warburton's [27] method and taking the dimensions(a/R, h/R) of ref.[13] for circular cylindrical shell with simply supported ends. The natural frequency (Ω)versus subtended angle(θ) at center of a cylindrically curved panel with the SSSS boundary condition and the same a/R and h/R ratios as that of full cylindrical shell is shown in Figure 4b. It is evident that for a given a/R and h/R ratio, one subtended angle is obtained with the natural frequency Ω =0.2516 being lowest [18],[19],[24]. The corresponding subtended angle is π/18 radians (optimum periodic angle θ_0) and oscillates in the r=1(first axial) and s=1(circumferential mode). The corresponding panel with subtended angle π/18 radians(10 degrees) is called the optimum periodic curved panel. However, in reference [11]-[13], the periodic unit cell (curved plate) angles are presumed to be aleatory i.e. π/22 radians and π/33 radians.

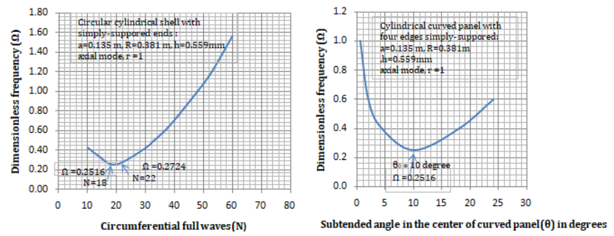
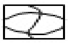
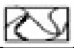


Figure 4. (a) Radial free vibration frequency(Ω) (a) versus circumferential full waves(N) of full circular cylindrical shell using Warburton approach [19],(b) versus subtended angle(θ) in the center of cylindrically curved panel[19]

The findings of the boundary frequencies (Ω) for a curved panel with the ideal(optimum) periodic angle θ_0 = π/18(optimum) are shown in Table 2 for the first propagation band and first two axial modes(r= 1, 2).The minimum frequency Ω=0.2516 is determined, which corresponds to the curved panel with four edges simply-supported(SSSS) boundary conditions and vibrating in first axial(r=1) and circumferential(s=1) mode. There is a coherent gap between Ω =0.2516 (θ_0 = π/18 radians, optimum) and Ω =0.2724 (θ =π/22 radian). From the discussion above, it can be inferred that if the chosen periodic curved panel angle is π/22 radian [11]-[13] for this particular geometry, the lowest bounding frequency (fundamental) Ω = 0.2516 is missed.

The bounding frequency results are compared with published literature data[19,24] and found good agreement. The modes of bounding frequencies are identified and shown for first two axial modes(r=1,2) in Table 2.

Table 2. Comparison of BF(Ω)of phase-frequency curve or first propagation band(PB I)for optimum periodic angle($\pi/18$ radians) and r=1,2 with references[19,24].

	Dimensionless bounding frequencies(Ω) and modes of propagation bands(PB I) at phase constants ϵ_y=0 and ϵ_y=π	
r	$\epsilon_y = 0$	$\epsilon_y = \pi$
1	0.5854+ (0.5840) 0.5880* s=2, (SSSS) 	0.2516+ (0.2516) 0.2526* s=1, (SSSS) 
2	0.7062+ 0.7089* s=2,(SSSS) 	0.5412+ 0.5402* s=1,(SSSS) 

(+Upper values are present approach; Bracket values are reported in[19]; *values are frequencies obtained in [24] corresponding to optimum periodic angle.Lower values are modes(r, s)in the axial and circumferential direction of the single curved panel with SSSS edges boundary conditions[24]). Further, the bounding frequency can be generated for different periodic angles. However, for a particular geometry (a/R,h/R) of shell, one can not get the lowest fundamental frequency for other periodic angles(θ) except $\theta_0=\pi/18$ i.e. 10 degrees(optimum periodic angles). The natural frequency versus subtended angle in the center of curved panel cross section is depicted in Figure 4b with four edges simply supported. One can get the boundary frequency(lower point of PB I as shown in Figure 3, at $\epsilon_y=\pi$) for other periodic angles using Figure 4b graph. However, these are not significant for the design and analysis of cylindrical shell structures.

4 Conclusions

A new formulation of the wave propagation in a multi circumferential LS supported cylindrical curved panel or a complete shell in bending vibrations is described in this study using an approximate solution. The periodic structure theory in a wave approach is used to describe the wave motion in circumference of cylindrical shell with periodic circumferential line supports. Plane wave motion type has been considered. Consequently, only simply propagating waves are considered with no attenuation. Bounding modes of a periodic beam's propagation bands are coupled with basic beam functions to generate displacement functions that satisfy Floquet's theory. Derive the stiffness and mass matrices of periodic unit cell (single periodic curved panel) using the Rayleigh-Ritz approach. To find the phase-frequency relation, use the aforementioned technique to solve the eigenvalue problem. The bounding frequencies and bounding modes results of the current beam function with periodic structure formulations are well comparable to those of the literature-available hierarchical functions. This validates the proposed present formulation. The benefit of this approach is that only small-order matrices need to be examined. To identify the vibration modes, these bounding frequencies from the current formulation are compared with single curved panel free vibration dimensionless frequencies with various edge boundary conditions. Next, the findings of the boundary frequencies for the periodic curved panel's with an optimum angle which is the lowest frequency for the specified cylindrical shell geometry (a/R, h/R) have been demonstrated. The limitation of present work is the type of wave considered does not provide information about attenuated (or complex conjugate) waves. This is because ϵ_y is assumed to be purely imaginary. The objective of future study is to extend the current methodology for determining the propagation surfaces of an orthogonal grid of a line-supported cylindrical shell.

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6 Author contribution statements

The formulation, design, literature review, and study of the results, as well as the article's spelling and content checking, were all completed by the author.

7 Ethics committee approval and conflict of

8 interest statement

"Article prepared does not require approval from the Ethics Committee". "Article prepared has no conflict of interest with any person/institution".

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