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FRACTAL DIMENSIONING OF SAND GRAINS USING IMAGE ANALYSIS SYSTEM

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ABSTRACT

Engineers and earth scientists have successfully used the concept of fractal theory to better analyze the roughness of soil and/or rock particles, and how it affects the permeability, structure and distribution of pores in sedimentary rocks and their influence on strength. Use of fractals as a way to describe irregular or rough objects has been highlighted in articles by researchers working in fields such as powder mechanics, rock and soil mechanics, sedimentary petrography and geoenvironmental applications. Fractal scaling has been found appropriate to express such scale independence for collection of soil particles and aggregates. In many aspects, soil is a fractal medium and fractal models are available for the fragmentation of aggregates with fractal pore space, and with fractal surface. Applications of fractal concepts encompass description of soil physical properties such as pore-size distribution, pore surface area, and grain-size distribution. The roughness of particulate soils is an important characteristic that affects the mass behavior of the soil. The area-perimeter technique was used to predict the fractal dimension using image analysis system. This paper presents the effects of the roughness and sorting of the sand patterns with different shapes on fractal dimension. Results confirmed the significance of the roughness effect on fractal dimension.

Key Words : Sand grains, Fractal dimension, Area-perimeter method, Image analysis

GÖRÜNTÜ ANALİZİ İLE KUM TANELERİNİN FRAKTAL BOYUTUNUN BELİRLENMESİ

ÖZET

Mühendisler ve yer bilimciler, hidrolik iletkenlik, zemin yapısı, boşluk dağılımı veya bunların zemin dayanımına etkisini, zemin ve kaya tanelerinin pürüzlülüğünü araştırmak için, fraktal teori kavramını başarılı bir şekilde kullanmaktadırlar. Düzensiz ve pürüzlü parçaçıkları tanımlamak için fraktalların kullanımına; toz mekaniği, kaya ve zemin mekaniği, sedimentoloji ve çevre-geotekniği alanında çalışan araştırmacılar tarafından dikkat çekilmektedir. Fractal boyutlandırma zemin ve çakıl gruplarının ölçeksiz boyutlandırılması için uygun bulunmaktadır. Pek çok açıdan zemin fraktal bir ortamdır ve boşluk yapısına sahip zemin taneleri için fraktal modeller geliştirilmiştir. Fraktal yaklaşım uygulamaları zeminlerin tane çapı dağılımı, zemin boşluklarının alanı ve dağılımı gibi fiziksel özelliklerin tanımlanmasını kapsamaktadır. Taneli zeminlerin pürüzlülüğü zeminlerin kütlesel davranışına etki eden önemli bir özelliktir. Bu çalışmada, görüntü analiz sistemi kullanılarak kum tanelerinin fraktal boyutu alan-çevre metodu ile belirlenmiştir. Farklı şekillerdeki kum tanelerinin fraktal boyutu belirlenmiş ve pürüzlülüğünün fraktal boyuta etkisi araştırılmıştır.

Anahtar Kelimeler : Kum taneleri, Fraktal boyut, Alan-çevre metodu, Görüntü analizi

1. INTRODUCTION

Fractals are relatively new mathematical concept for describing the geometry of irregularly shaped objects in terms of frictional numbers rather than integer. The concept of fractals introduced by Mandelbrot (1967), which has the shape formed in nature, has been usually analyzed using euclidian geometry. The key parameter for fractal analysis is the fractal dimension, which is a real non-integer number, differing from the more familiar euclidean or topological dimension. The fractal dimension for a line of any shape varies between one and two, and for a surface between two and three. Recently, engineers and earth scientists have successfully used this concept to better analize the roughness of particles; distribution of rock fragments resulting from blasting; interconnectivity and distribution of fissures in geological materials, and how this affects their permeability; the texture and distribution of pores in sedimentary rocks and their influence on permeability and strength; as well as many other geological materials (Vallejo, 1997). Using of fractals as a way to describe irregular objects has been highlighted in articles by researchers working in fields such as powder (Kaye, 1978), rock (Carr and Warriner, 1989; Ghosh and Doeman, 1993) and soil mechanics and geoenvironmental engineering (Vallejo, 1996, 1997). Fractal scaling has been found appropriate to express such scale independence for collections of soil particles and aggregates (Bartoli et al., 1992; Perfect and Kay, 1995; Young and Cramford, 1991), porosity (Bartoli et al., 1991) and soil pore surfaces. Soil is a fractal medium by many aspects and fractal models are available for the fragmentation of aggregates with fractal pore space and fractal surface. In each case, the aggregates are composed of building blocks of finite size (Perfect, 1997). Applications of fractal concepts encompass description of physical soil properties such as pore-size distribution (Brakensiek et al., 1992), pore surface area (Friesen and Mikula, 1987), and particle-size distribution (Tyler and Wheatcraft, 1992; Wu et al., 1993).

Fragmentation caused by the propagation of multiple fractures at different length scales. Such fractures can be induced by dynamic crack growth during comparative loading or by stress waves during impact loading. Fractal theory, which deals with the scaling of hierarchical and irregular systems, offers new opportunities for modeling the fragmentation process (Perfect, 1997). While fractal models for the fragmentation of rocks and soils are relatively well developed, their experimental verification is less realized or entirely lacking. In fact, there is no clear experimental evidence about the nature of the fractal dimensions. Further research is needed to evaluate the predictive capabilities of fractal dimensions determined independently (Gimenez et al., 1997). For this reason in this paper, different sand samples which have different shape and roughness, were analyzed to predict the roughness fractal dimensions by image analysis method, in order to examine the effect of the roughness of the sand patterns on the roughness fractal dimension and to evaluate the applicability and usefulness of the dimensioning technique.

2. IMAGE ANALYZING OF SAND GRAINS

Three sand types with different size and texture were used in experiments for obtaining fractal dimension. Two of them are Ottawa sands and the third one is cracked sample taken from compacted material in laboratory. Sand grains were photographed by using a computerized image analysis system and parameters were calculated by image analyzing program, BIOQUANT. The analog photographic images were digitized into numeric representations within the BIOQUANT program by segmenting the analog image into a binary image. Once in a binary code, the perimeter and area for each individual sand profile were determined by means of BIOQUANT. These parameters were plotted for each sand types and Dr was determined for sand types from the slope of the best-fit line in Figure 2.

Figure 1 shows images of the three sand types. All sand samples were magnified 28 times than their true size, due to magnifying limit of the equipment. Based on imaged samples, it is apparent that the two-dimensional sand profiles that have a greater average fractal dimension if they have higher roughness (Vallejo, 1997). Figure 1a and 1b show uniform Ottawa sands and Figure 1c. shows a fractured sand grain under compaction tests. We can see that sample in Figure 1c has larger fractal dimension than Ottawa sands, because of its higher roughness. The resulting analysis of D_r primarily reflected the textural aspects of the sand profile since the magnification of the viewing image was relatively high.

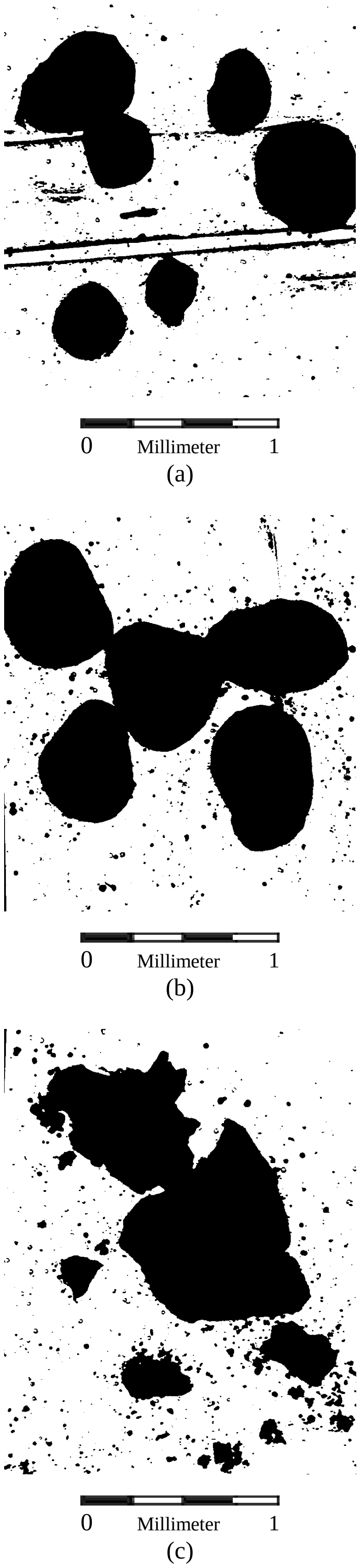

Figure 1. Image analyzing of individual sand grains; a) and b) Ottawa sands which have different grain diameters, and c) fractured sand

3. FRACTAL DIMENSION OF SAND GRAINS

3. 1. Fractal Scaling For Sands And Rocks

Mandelbrot (1977) developed fractal geometry to characterize the irregular and fragmented patterns within nature. This approach provided a means to study the roughness and distribution parameters. Soil particle roughness and size distribution often considered to be irregular (rough or fragmented) properties. Mathematical fractals have been used in modelling the geometry of porous media. The fundamental equation applied to all fractals is the number-size relationship (Mandelbrot, 1983; Feder, 1988):

$$N(r) = kr^{D} \qquad (1)$$

Where N(r) is the number of elements with radius equal to r, k is the number of initiators of unit length and D is the fractal dimension (the mass, the volume, the roughness, esc.). Equation (1) is used in its cumulative form by replacing N(r) by the number of cumulative elements of length greater than or equal to r. To find the fractal dimension we can get logarithm of the Equation 1.

$$D = [\text{Log } N(r) - \text{Log } k]/\text{Log } (1/r) \qquad (2)$$

Fractal dimensions can be easily obtained from the fractal models, which are Cantor bar, Koch curve, Sierpinski carpet, and Menger sponge (Gimenez et al., 1997), assuming k = 1. Several different fractal dimensions can be used to characterize the geometry of a porous medium. Fractal dimension of aggregates mass, pore volume, pore surface, roughness and size distribution of fragments have all been observed for geological materials such as rocks and soils. These fractals are follows:

A fractal dimension of mass, D_m, quantifies space-filling characteristics of the solid in a space of radius r. For a mass fractal, scaling of mass, M, follows a relationship of form:

$$M = r^{Dm} \qquad (3)$$

Where D_m is the fractal dimension of mass. This Equation can also be applied to separate entities (e.g. soil aggregates) of different radii but similar shape (Gimenez et al., 1997).

A fractal dimension of pore volume, D_v, quantifies the space filling properties of pore volume, V, in space of radius r. Scaling of pore volume follows

$V=r^{D_v}$ (4)

Where D_v is the fractal dimension of pore volume. In Equations 3 and 4, M and V are similar in form, but represent scaling of complementary properties, ie. Mass and pore space (Avnir et al., 1985; Bartoli et al., 1992; Van Damme, 1995).

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By evaluating the ratio of linear extents, which can be represented by either the length (L), the square-root of area ($A^{1/2}$), and the cube-root of volume ($V^{1/3}$), within a population of geometrically similar fractal shapes, a fractal dimension can be developed. It can be seen that the ratio of any two of these linear extents will give a constant value specific to the pattern. For instance, consider the Euclidean shape of a circle and square, each with linear extents represented as the perimeter (P) and square-root of area ($A^{1/2}$). For a circle with radius, r.

$C_c=P/A^{1/2}$ (5)

Where $C_c=2\pi^{1/2}$ for all circles. Therefore, the ratio of linear extents for a particular population of geometrically similar shapes is constant. By evaluating the ratio of linear extents within a population of geometrically similar fractal shapes, a fractal dimension can be developed (Vallejo, 1997). Mandelbrot (1983) proposed that the "ratio of linear extents" of fractal patterns are in themselves fractal, and that

$c= ((P)^{1/Dr})/ (A)^{1/2}$ (6)

or

$A=cP^{2/Dr}$ (7)

Where c is a constant for similar fractal shapes and D_r is the roughness fractal dimension of the population. Again, taking the logarithm of Eq.7, c yields a linear relationship between area a and perimeter P with Dr related to the slope coefficient, m, by:

$D_r = 2/m$ (8)

Evaluating the roughness fractal dimensions D_r using Eq. 8, $D_r = 2/m$ is referred to as the "area-perimeter" method which determines D_r by evaluating an entire population of related shapes as opposed to individual particles.

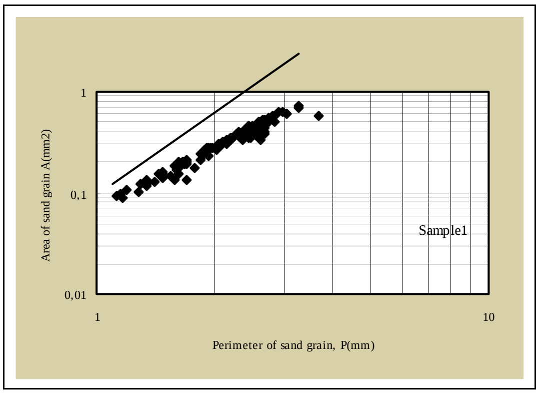
3. 2. Results and Discussion

In the present study, the area-perimeter fractal method used to measure the roughness fractal dimension of sand particles. The area-perimeter method derives D_r based on a single level of scrutiny, i.e. a single measurement of area and perimeter. Therefore, evaluating D_r with respect to textural and structural fractals must be performed by evaluating the level of scrutiny of measurement with respect to the object being dimensioned. If the level of particle magnification is very high with respect to the overall shape of the object (high viewing magnification), the D_r will reflect the textural characteristics of the object (Vallejo, 1997). Similarly, if the level of scrutiny is low relative to the overall size the object (low viewing magnification), the D_r will reflect structural aspects.

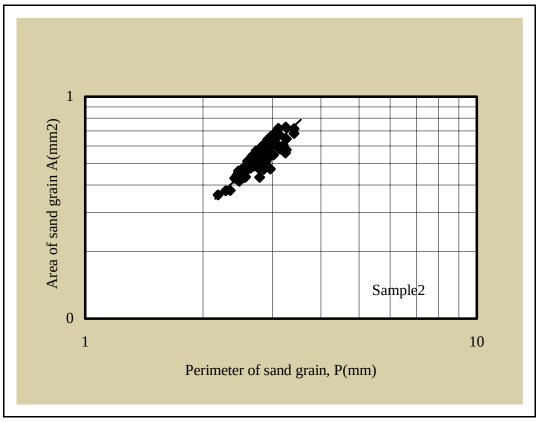
Many individual sand grains were measured with the image analysis system. Table 1 contains area-perimeter fractal dimensions for the three types of sand analyzed in this research. Fractal dimensions were significantly affected by the particle roughness and size. The lowest D_r corresponded to the sample 1 and the larger D_r corresponded to the sand 3, based on visual inspection. Areas and perimeters for individual sand grains obtained from image analysis system of sand types are plotted in Figure 2. Fractal dimensions Dr for sands derived from Eq. 8 using the slope obtained form the area (log A) versus perimeter (log P) plot. The image of all sands (sample 1, 2, and 3) were obtained at relatively high levels of magnification and, therefore, the area-perimeter Dr reflects the textural aspects of the profiles of sands.

Table 1. Results of Fractal Scaling From Area and Perimeter Method by Using Image Analysis System

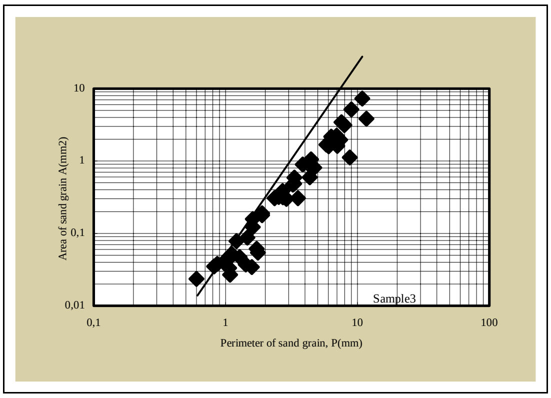
Sand Type	Number of Individual Grains Analyzed	Average D_r	Standard Deviation	Aspect Ratio
Sand 1	111	1.042	0.159	Texture
Sand 2	160	1.151	0.101	Texture
Sand 3	48	1.776	0.477	Texture



(a)



(b)



(c)

Figure 2. Roughness fractal dimensions and areas - perimeters from individual sand grains obtained from Image Analyzing System. a) and b) Ottawa sands which have different grain diameters, and c) fractured sand.

4. CONCLUSIONS

The following general conclusions were drawn from the interpretation of the test results.

- Sand grains originated from three different sand types were photographed by using a computerized image analysis system and parameters were calculated by image analyzing program, BIOQUANT.
- The area-perimeter method was applied herein using data obtained from the use of an automated image analysis system on magnified profiles.
- The results indicate that the roughness fractal dimension, D_r, is directly related to the roughness and size of individual sand grains in that the rougher the sand grain profile, the higher the measured D_r. The lowest D_r corresponded to the sample 1 and the larger D_r corresponded to the sand 3, based on visual inspection.
- It is important to consider the level of magnification under which the fractal dimensions are derived so as to differentiate between textural and structural aspects of roughness. Therefore, further research is needed to evaluate the predictive capabilities of textural and structural aspects of form.

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6. NOTATIONS

A = area
c = constant for similar fractal shapes
c_c = constant for circular shapes
D = the fractal dimension
D_m = fractal dimension of mass
Dr = roughness fractal dimension of curve
D_v = fractal dimension of pore volume
k = the number of initiators of unit length
M = mass of fractal shape
m = slope coefficient
N(r) = the number of elements
P = perimeter
R = Curve of fractal shape
r = radius of fractal shape
V = pore volume of fractal shape

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