



# IMPLEMENTATION OF OPTIMUM RESOURCE ALLOCATION BY FUZZY GOAL PROGRAMMING : THE CASE OF HIGHER EDUCATION SYSTEM

**Mustafa GÜNEŞ\*, Nurullah UMARUSMAN\*\***

\*Eastern Mediterranean Univ., Faculty of Engineering, Dep. of Industrial Engineering, G. Magusa/TRNC-Mersin-TR

\*\*University of Dokuz Eylül, Faculty of Economic and Administrative Sciences, Department of Econometrics, Buca/Izmir

Received Date : 14.11.2002

## ABSTRACT

The Goal Programming, which is using to solve the multiple objective decision problems, has wide and great potential among other methods targeting maximization or minimization of goals. The main aim of the goal programming is to minimize the biases from each objective, instead of optimization of goals. Goal Programming algorithms, as originally developed by Charnels, attempts to achieve as many of these goals possible by minimizing deviation variables from the goal levels, depending on their relative weights. This minimization process has been forming in two categories, which involves preemptive and weighted techniques. In this study, Fuzzy Goal Programming has used to determine optimum allocation of education equipment such as computer and laptop to the faculty members and officers at different level of positions.

**Key Words :** Goal programming, Fuzzy goal programming, Optimization

## BULANIK AMAÇ PROGRAMLAMA YARDIMI İLE OPTIMUM KAYNAK TAHSİSATI : YÜKSEK EĞİTİM SİSTEMİ UYGULAMASI

## ÖZET

Çok amaçlı karar problemlerinin çözümü için kullanılan Amaç Programlama , enbüyükleme veya enküçükleme maksadı ile kullanılan diğer metodlar arasında daha kapsamlı ve büyük potansiyele sahiptir. Amaç programlamanın esas gayesi, hedeflerin optimizasyonu yerine, hedeflerden sapmaları enküçüklemektir. Charnels tarafından geliştirilen Amaç Programlama, farklı önem seviyelerine sahip birden çok amacın, minimum sapmalarla, aynı anda gerçekleşmesini hedeflemektedir. Bu enküçükleme tekniği, önceliklendirme ve ağırlıklandırma olmak üzere iki kategoride uygulanmaktadır. Bu çalışmada, Bulanık Amaç Programlama tekniği, bir fakültede çalışan personele dağıtılması düşünülen bilgisayar,yazıcı, tarayıcı gibi eğitim araçlarının, bütçe ve makine sayısı kısıtları dikkate alınarak, en iyi tahsisini belirlemek için kullanılmıştır.

**Anahtar Kelimeler :** Amaç programlama, Bulanık amaç programlama, Optimizasyon

## 1. INTRODUCTION

The Goal Programming (GP) Technique is very useful tool for decision makers to discuss many targets in finding a set of suitable and acceptable

solutions of decision problems. Due to its above characteristics, many decision problems of top managers have been solved so far. However, determining precisely the goal value of each objective is difficult for decision maker, since possibly only partial information can be obtained (Ling-Hsuan and Feng-Chou, 2001). Since Zadeh

proposed the concept of fuzzy sets, Bellman and Zadeh have developed a basic framework for decision-making in a fuzzy environment (Hasio-Fan and Ching-Chun, 1977). There after, many researches followed in which Narasimhan and Hannan have extended the fuzzy set theory to the field of goal programming.

The other typical application of deterministic multi-objective programming models have been applied for planning solid waste management systems. For example, Per lack and Willis considered the application of a multi-objective programming model in a sludge disposal problem in the USA. Koo et al. (1991) accomplished the sitting planning of a regional hazardous waste treatment center by using a fuzzy multi-objective programming technique in Korea (Ni-Bin and Wang, 1997), and Fuzzy Goal Programming Approach for Water Quality Management in a River Basin has published in Fuzzy Sets and Systems (Chih- Sheng and Ching-Gung, 1997). M.Arenas Parra, A.Bilbao Terol, M.V. Rodriguez Uria have discussed a Fuzzy Goal Programming Approach to Portfolio Selection (M-Para and A-Terol and M-Uria, 2000), Liang-Hsuan Chen, Feng-Chou Tsai have formulated fuzzy goal programming (FGP) incorporating different importance and preemptive priorities by using an additive model to maximize the sum of achievement degrees of all fuzzy goals (Ling-Hsuan and Feng-Chou, 2001). T.K.Roy and M.Maiti have discussed Multi-Objective Inventory Models of Deteriorating Items With Some Constraints In Fuzzy Environment (Roy and Maiti, 1998) and Jong Soon Kim and Kyu-Seung Whang have investigates the application of tolerance concepts to goal programming in a fuzzy environment (Jong-Soon and Kyu- Seung, 1998).

Three methodologies capable of effectively dealing with multi-objective programming problems are vector maximum (VM) methods, goal programming (GP) approaches and interactive techniques (Chih- Sheng and Ching-Gung, 1997). Every approaches has their own advantages and disadvantages depending on their structures. Briefly, VM method has the advantages of variety of alternatives they yield. GP approaches have directly arrived at an acceptable compromise solution. The interactive techniques are more often desirable because they yield a single preferred solution. A disadvantage of those three methods is their strong dependence upon local information that occasionally cannot arrive at an “optimal” solution.

Goal Programming is a decision tool in modeling real world decision problems that has been extensively used in solving decision-making problems, especially involving multiple conflicting

goals. In solution procedure, it is necessary to determine aspiration levels for the objectives that can be rank ordered, depending on their significance to the decision maker. Goal Programming algorithms attempt to achieve as many of these goals possible by minimizing deviation variables from the goal levels, depending on their relative weights. However, a major limitation of GP is that the aspiration level and / or priority factors are imprecise in nature for the decision maker. Under such a circumstance, using the fuzzy set theory allows vague aspirations of the decision maker to be quantified and is used in a decision making problem (Chih- Sheng and Ching-Gung, 1997).

There is quite difference between Goal Programming and Fuzzy Goal Programming such as follows; Goal Programming requires the decision maker (DM) to set definite aspiration values for each objective that he/she wishes to achieve, whereas the latter is specified in an imprecise manner. A fuzzy goal is considered here as a goal with an imprecise aspiration level. Consideration of different relative importance and priorities of the goals in the Fuzzy Goal are proper than others. Narasimhan has used linguistic variables, such as “very important”, “less important” and “moderately important”, to describe the fuzzy weights of the goals, and defined the corresponding membership functions by specifying the desirable intervals of membership degree to reflect the importance (Narasimhan, 1980).

## 2. MODELS OF THE FUZZY GOAL PROGRAMMING

Programming is important because some of the goals are less or more important A classical structure of the multi-objective programming model is as follows.

$$\begin{aligned} & \text{Max } Ax \\ & \text{s.t. } Cx \leq d, \\ & \quad \quad \quad x \geq 0 \end{aligned} \tag{1}$$

Where  $x$  is an  $(nx1)$  alternative set,  $A$  is an  $(m \times n)$  matrix of coefficients of objective functions,  $C$  is a  $(pxn)$  matrix of coefficients of constraints and  $d$  is a  $(px1)$  right-hand side values of model.

The model (1) can be reformulated as a Fuzzy Goal Programming problem, in the case of presentation of fuzzy information in which the aspiration level set  $b_0$  can be constructed by using the pay-off table, such as below;

Find  $x$   
 s.t.  $Ax \cong b_0$  (or  $\tilde{b}$ )  
 $Cx \leq d, \dots$  (2)  
 $x \geq 0,$

Where both the symbol  $\cong$  (or  $\tilde{b}$ ), express linguistic goals such as “the profit should be around  $b$ ”.

In addition to above alternatives, if it is possible to consider the RHS values as a fuzzy (fuzzy resources), the model (2) can be written as

Find  $x$   
 s.t.  $Ax \cong b_0$  (or  $\tilde{b}$ )  
 $Cx \leq d,$  (3)  
 $x \geq 0,$

Where the fuzzy equality constraints express that earned profits should be around  $b_i$ , and the symbol  $\leq$  indicates the fuzziness of the constraint and is read as “approximately less than or equal to”. The equation (2) and (3) can be solved by similar methods if similar membership functions are used for modelling the imprecise nature of “fuzzy goals” and “fuzzy equality”, (Chih- Sheng and Ching-Gung, 1997).

To resolve these fuzzy equalities, we should elicit their membership functions based on preference concept from the Decision Maker(s). For the aim of mathematical tractability, the membership function  $\mu_i(x)$

$$\mu_i(x) = \begin{cases} [(Ax)_i - (b_i - d_i)]/d_i & b_i - d_i \leq (Ax)_i < b_i \\ [(b_i - d_i) - (Ax)_i]/d_i & b_i \leq (Ax)_i \leq b_i + d_i \\ 0 & b_i + d_i < (Ax)_i \text{ or } (Ax)_i < b_i - d_i \end{cases} \quad (4)$$

To solve Equation (3), when Equation (4) is given, Narasimhan proposed the following  $2^m$  sub-problems of equivalent standard linear goal programming:

$$\max \{ \min_i [(Ax)_i - (b_i - d_i)]/d_i \}$$

such that  $b_i - d_i \leq (Ax)_i \leq b_i + d_i$  ..... (5)  
 $x \geq 0$

and

$$\max \{ \min_i [(b_i - d_i) - (Ax)_i]/d_i \}$$

such that  $b_i \leq (Ax)_i \leq b_i + d_i$  ..... (6)  
 $x \geq 0$

By putting equations (5) and (6) together, we obtain

$$\max \alpha$$

s.t  $[(Ax)_i - (b_i - d_i)]/d_i \geq \alpha$   
 $b_i - d_i \leq (Ax)_i \leq b_i$   
 $[(b_i - d_i) - (Ax)_i]/d_i \geq \alpha$  ..... .. (7)  
 $b_i \leq (Ax)_i \leq b_i + d_i$   
 $\alpha \in [0,1] \text{ ve } x \geq 0$

Moreover, the first two constraints of equation (7) can be expressed as

$$\alpha \leq 1 + [(Ax)_i - b_i]/d_i \text{ and}$$

$$(b_i - d_i)/d_i \leq (Ax)_i/d_i \leq b_i/d_i \text{ ..... } \dots (8)$$

let us  $(Ax)_i/d_i = b_i/d_i - \delta_i^+$ , where  $\delta^+$  is overestimated; and we have;

$$\alpha + \delta_i^+ \leq 1 \text{ ve } (Ax)_i/d_i - \delta_i^+ = b_i/d_i \text{ ..... } (9)$$

Similarly, third and fourth constraints of Equation (7) are equivalent to :

$$\alpha + \delta_i^+ \leq 1 \text{ and } (Ax)_i/d_i - \delta_i^+ = b_i/d_i \text{ .....} (10)$$

Hannan then combined Equation (9) and (10) to obtain the following linear goal programming model which is equivalent to Narasimhan  $2^m$  sub-problems (Hannan, 1981):

$$\max \alpha$$

s.t  $(Ax)_i/d_i + \delta_i^- - \delta_i^+ = b_i/d_i$   
 $\alpha + \delta_i^- + \delta_i^+ \leq 1$   
 $\delta_i^-, \delta_i^+ \geq 0$   
 $\delta_i^- \delta_i^+ = 0$   
 $\alpha \in [0,1] \text{ and } x \geq 0 \text{ ..... } \dots (11)$

on the other hand, Yang, Ignizio, and Kim used Zimmermann’s fuzzy programming to solve (2) with membership functions of (4) and obtained the following auxiliary model :

$$\max \alpha$$

$[(Ax)_i - (b_i - d_i)]/d_i \geq \alpha$   
 $[(b_i + d_i) - (Ax)_i]/d_i \geq \alpha$  (12)  
 $\alpha \in [0,1]$

if the deviations  $d_i$  from the centers  $b_i$  are different, we would have the following model:

$$\max \alpha$$

$[(Ax)_i - (b_i - d_{i1})]/d_{i1} \geq \alpha$  (13)  
 $[(b_i + d_{i2}) - (Ax)_i]/d_{i2} \geq \alpha$

Comparison of equation (7), (11) and (12) with isosceles triangular membership function are given below Table1.

Table 1. Comparasion of Alternative Models

Equation No	LP No	Constraints No	Variables
7	2 <sup>m</sup>	3m	n+1
11	1	2 m	n+2m+1
12	1	2 m	n+1

### 3. CASE STUDY OF HIGHER EDUCATION ALLOCATION PROBLEM

In this section of the study, we are using the real allocation problem of a certain higher education institute that top manager would like to distribute or allocate the educationally equipments to the member of faculty and officers. Still, there are five different teaching media to be bought like; desktop computer, laptop computer, printer, Cdwriter and scanner with the limited budget resource of \$120.000. Each equipment`s prices are as follows:\$1100, \$2050, \$320, \$250, \$180 respectively.

In addition to above restrictions, top management would like to determine the optimum number of each equipment,as 55 desktops, 28 Laptops, 18 Printers, 13 Cdwriters and 10 Scanners. On the other hand, there are a restrictions of maximum bias of \$10.000 for budget and biases of 7 desktops, 4 Laptops, 4 Writers, 3 CD Writers and 3 Scanners.

Formulation of the decision problem under the restrictions of the budged possibilities of the Institute is listed below:

$$g_1(x) = 55X_1 + 28X_2 + 18X_3 + 13X_4 + 10X_5 = 120000$$

$$g_2(x) = X_1 = 1100$$

$$g_3(x) = X_2 = 2050$$

$$g_4(x) = X_3 = 320$$

$$g_5(x) = X_4 = 250$$

$$g_6(x) = X_5 = 180$$

#### 3. 1. Solution of the problem with Fuzzy Goal Programming

Triangular membership functions for each one;

$$\mu_1(g_1(x)) = (120.000, 108.000, 132.000)$$

$$\mu_2(g_2(x)) = (1100, 900, 1300)$$

$$\mu_3(g_3(x)) = (2050, 1950, 2150)$$

$$\mu_4(g_4(x)) = (320, 275, 365)$$

$$\mu_5(g_5(x)) = (250, 235, 265)$$

$$\mu_6(g_6(x)) = (180, 160, 200)$$

The mathematical structure of the model in according to the (11) equation;

Max  $\alpha$

$$55X_1 + 28X_2 + 18X_3 + 13X_4 + 10X_5 + 12.000\delta_1^- - 12.000\delta_1^+ = 120.000$$

$$X_1 + 200\delta_2^- - 200\delta_2^+ = 1100$$

$$X_2 + 100\delta_3^- - 100\delta_3^+ = 2050$$

$$X_3 + 45\delta_4^- - 45\delta_4^+ = 320$$

$$X_4 + 15\delta_5^- - 15\delta_5^+ = 250$$

$$X_5 + 20\delta_6^- - 20\delta_6^+ = 200$$

$$\alpha + \delta_1^- + \delta_1^+ \leq 1$$

$$\alpha + \delta_2^- + \delta_2^+ \leq 1$$

$$\alpha + \delta_3^- + \delta_3^+ \leq 1$$

$$\alpha + \delta_4^- + \delta_4^+ \leq 1$$

$$\alpha + \delta_5^- + \delta_5^+ \leq 1$$

$$\alpha + \delta_6^- + \delta_6^+ \leq 1$$

$$\delta_1^-, \delta_1^+ \geq 0$$

$$\delta_i^- \delta_i^+ = 0, \quad i=1,2,3,4,5,6$$

$$\alpha \in [0,1] \text{ and } x \geq 0$$

The structure of the model in according to the equation (12) is as follows:

Max  $\alpha$

$$[132000 - (55X_1 + 28X_2 + 18X_3 + 13X_4 + 10X_5)] / 12.000 \geq \alpha$$

$$[55X_1 + 28X_2 + 18X_3 + 13X_4 + 10X_5 - (108.000)] / 12.000 \geq \alpha$$

$$[1300 - (X_1)] / 200 \geq \alpha$$

$$[X_1 - 900] / 200 \geq \alpha$$

$$[2150 - X_2] / 100 \geq \alpha$$

$$[X_2 - 1950] / 100 \geq \alpha$$

$$[365 - X_3] / 45 \geq \alpha$$

$$[X_3 - 275] / 45 \geq \alpha$$

$$[265 - X_4] / 15 \geq \alpha$$

$$[X_4 - 235] / 15 \geq \alpha$$

$$[200 - X_5] / 20 \geq \alpha$$

$$[X_5 - 160] / 20 \geq \alpha$$

$$\alpha \in [0,1] \text{ and } \dots$$

The formulation of the decision problem with fuzzy goals has completed and became ready to solve. As previously stated that the more important side of the problem is the goal and higher desirable achievement degree. Conventional linear programming or integer programming can solve the above-developed model, but integer programming algorithm solution could be more logical since it has integer values. Solution of the fuzzified model developed in according to the above (12) system and normal GP model have yielded the results listed at the Table 2.

Table 2. Alternative Solutions of Fuzzified Models

Results of Fuzzy GP Problem	Results of GP Problem
X <sub>1</sub> = \$1080.828	X <sub>1</sub> = \$1053.454
X <sub>2</sub> = \$2040.414	X <sub>2</sub> = \$2050
X <sub>3</sub> = \$315.6862	X <sub>3</sub> = \$320
X <sub>4</sub> = \$248.5621	X <sub>4</sub> = \$250
X <sub>5</sub> = \$178,0827	X <sub>5</sub> = \$180
X <sub>6</sub> =0.9041378 (α value)	

The α value that we do try to determine is the maximum likelihood degree of the problem, which is, maximizes the efficiency of allocation facility. It has considered as a variable of X<sub>6</sub> inside of the model.

On the other hand, other solution of the justified model of the same problem could be more significant and helpful for decision makers, which is determine the optimum number of equipment, will be distributed. Under the circumstances of this idea, the output of second solution that has a preemptive priority for achieving goals, found such as following Table 3.

Table 3. Solution of Preemptive Priority

Results of Fuzzy GP Problem	Results of GP Problem
X <sub>1</sub> = 54(Optimum number of Desktop)	X <sub>1</sub> = 55 Desktop
X <sub>2</sub> = 27 (Optimum number of Laptop)	X <sub>2</sub> = 23 Laptop
X <sub>3</sub> = 9 (Optimum number of Printer)	X <sub>3</sub> = 18 Printer
X <sub>4</sub> = 9 (Optimum number of CD writer)	X <sub>4</sub> = 13 CD Writer
X <sub>5</sub> = 6 (Optimum number of Scanner)	X <sub>5</sub> = 10 Scanner

The necessary expenses may obtain by replacing numeric values of each decision variables to the budgeted constraint at the value of α membership level, as follows:

$$\$1080*(54) + \$2040.414*(27) + \$315.68*(9) + \$248.56*(9) + \$178*(6)= \$119956.16$$

The similar results has gathered from both (11) and (12) models mentioned in previous section of this study. As a final decision, one can offer the above optimal allocation policy to the top manager of

Institute or investor. But, by changing the relative importance of fuzzy goals, it is also possible to set other alternative solution of the same problem under the special conditions of restrictions.

#### 4. REFERENCES

- Chih-Sheng L., Ching- Gung, W. 1997. Fuzzy Goal Programming Approach for Water Quality Management in a River Basin, *Fuzzy Sets and Systems* 89, 181-192.
- Hannan, E. L. 1981. Linear Programming with multiple fuzzy goals, *Fuzzy Sets and Systems* 6, 235-248.
- Hasiao-Fan W., Ching-Chun F. 1997. A Generalization of Fuzzy Goal Programming With Preemptive Structure, *Computers Operations Research*, Vol. 24, No: 9, pp.819-828.
- Jong Soon, K., Kyu-Seung, W. 1998. The Tolerance Approach to the Fuzzy Goal Programming Problems with Unbalanced Triangular Membership Function, *European Journal of Operational Research* 107, 614-624.
- Koo et al., 1991. (Alınmıştır). Ni-Bin C., S. F. Wang, A. 1997. Fuzzy Goal Programming Approach for The Optimal Planning of Metropolitan Solid Waste Management Systems, *European Journal Of Operational Research* 99, 303-321.
- Liang-Hsuan C., Feng-Chou T. 2001. Fuzzy Goal Programming with Different Importance and Priorities, Theory and Methodology, *European Journal of Operational Research* 133, 548-556.
- M. Arenas P., A. Bilbao T., M. V. Rodriguez, U. 2001. A Fuzzy Goal Programming Approach to Portfolio Selection, *European Journal of Operational Research* 133, 287-297.
- Narasimhan, R. 1980. Goal Programming in Fuzzy Environment, *Decision Sciences* 11, 325-336.
- Ni-Bin C., S. F. Wang, A. 1997. Fuzzy Goal Programming Approach for The Optimal Planning of Metropolitan Solid Waste Management Systems, *European Journal Of Operational Research* 99, 303-321.
- Para, M., Terol, A., Uria, M. 2000. (Alınmıştır). Liang-Hsuan C., Feng-Chou T. 2001. Fuzzy Goal Programming with Different Importance and Priorities, Theory and Methodology, *European Journal of Operational Research* 133, 548-556.
- Roy, T. K. and Maiti, M. 1998. Multi-Objective Inventory Models of Deteriorating Items With Some Constraints in a Fuzzy Environment, *Computer Operations Research*, Vol. 25, No.12, pp. 1085, 1095.
- Young-Jou L., Ching-L. H.1994. Fuzzy Multiple Objective Decision Making, Springer-Verlag New York.