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Particle Swarm Optimization for Structural Design Problems

Yapısal Problemler Tasarımında Kuş Sürüsü Davranış Algoritması

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ABSTRACT

The aim of this paper is to employ the Particle Swarm Optimization (PSO) technique to a mechanical engineering design problem which is minimizing the volume of a cantilevered beam subject to bending strength constraints. Mechanical engineering design problems are complex activities which are computing capability are more and more required. The most of these problems are solved by conventional mathematical programming techniques that require gradient information. These techniques have several drawbacks from which the main one is becoming trapped in local optima. As an alternative to gradient-based techniques, the PSO does not require the evaluation of gradients of the objective function. The PSO algorithm employs the generation of guided random positions when they search for the global optimum point. The PSO which is a nature inspired heuristics search technique imitates the social behavior of bird flocking. The results obtained by the PSO are compared with Mathematical Programming (MP). It is demonstrated that the PSO performed and obtained better convergence reliability on the global optimum point than the MP. Using the MP, the volume of 2961000 mm3 was obtained while the beam volume of 2945345 mm3 was obtained by the PSO.

Keywords: *Particle swarm, Mechanical design, Design optimization.*

ÖZET

Bu makalenin amacı, makine mühendisliği tasarım problemlerinden olan bir ankastre kirişin belirlenen eğilme dayanımı sınır şartları içinde minimum hacmini hesaplayan bir Kuş Sürüsü Davranış Algoritması (Particle Swarm Optimization – PSO) uygulamaktır. Makine mühendislik tasarım problemleri çok karmaşık ve zaman alıcı hesaplamalar gerektirirler. Bu problemlerin çoğu geleneksel matematik hesaplamalarıyla türev alınarak çözümlenmektedirler. Problemlerin çözümlemeleri için türevlenebilir olmaları ve optimum noktanın bulunabilmesi için iyi bir başlangıç noktasından arama yapmaları gerekmektedir aksi taktirde global optimum yerine yerel optimum elde edilir. PSO Algoritması, geleneksel metotlara alternatif olarak türev gerektirmeyen ve global noktaya yakın bir noktadan arama yapma zorunluluğu olmayan doğadan esinlenerek seçim yapan bir metottur. PSO algoritması, kuşların kendi ve bağlı oldukları sürü ile bilgi alışverişi davranışlarından esinlenilerek geliştirilmiş popülasyon tabanlı bir optimizasyon tekniğidir. Bu çalışmada PSO Algoritması ile elde edilen sonuçlar Matematiksel Programlama (Mathematical Programming -MP) ile elde edilen sonuçlarla kıyas edilmiştir. Bu çalışmada PSO, global optimum noktayı bulmada yakınsama ve uygunluk bakımından MP den daha iyi olduğu gösterilmiştir. MP ile kirişin hacmi 2961000 mm3 bulunurken PSO ile kirişin hacmi 2961000 mm3 bulunmuştur.

Anahtar Kelimeler: *Kuş sürüsü davranış algoritması, Makine tasarımı, Tasarım optimizasyonu.*

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1. INTRODUCTION

The optimum design of mechanical engineering problems are complicated process due to a number of geometric parameters and constraints involved. Because of this complexity of the problems, the goal is to employ a practical procedure by which the mechanical engineering problems could be designed for an optimal solution. Advances in computers and computing techniques have proved to be a great chance to the world of optimization of engineering design problems. Although many gradient based techniques are available for optimization of engineering problems, they have several drawbacks from which the main one is becoming trapped in local optima. Among the latest heuristics optimization techniques, it can be seen the growing application of Particle Swarm Optimization (PSO), which is an approach of global optimization, in the literature (Ali and Kaelo, 2008). The PSO was developed in analogy upon simulation of social behavior of bird flocking by sharing information among their members in multiple dimensions in space looking for the best food source (global optimum) (Perez and Behdinan, 2007; Roy and Ghoshal, 2008). The PSO can provide a remarkable balance between exploration and exploitation of the search space. From this point of view, this study provides the use of the PSO to seek a global optimum solution to problem in hand.

2. THE PARTICLE SWARM OPTIMIZATION

The PSO was originally proposed and developed by Kennedy and Eberhart (1995). The PSO is an optimization technique motivated by social models of the animals such as swarm of birds or fish schooling (Ali and Kaelo, 2008) which search for best food sources in a very typical manner (Lee et al., 2008). The PSO algorithm is initialized with the population of candidate solutions, individual called particles, being randomly placed in the search space. Each individual of the swarm is assigned a velocity which is dynamically adjusted and updated according to the flying experiences of its own and companions (He and Wang, 2007). Each particle keeps track of its coordinate in the search space with a velocity, v_i, according to its personal previous best, p_{best}, solution and previous best solution of the entire swarm, g_{best}, to update the current position, x_i, of each particle in the swarm.

Depiction of the velocity and position updates is given in Figure 1. The position of each particle is updated by a new velocity calculated by the following formula:

$$x_{ij}(t) = x_{ij}(t-1) + v_{ij}(t) \quad (1)$$

$$v_{ij}(t) = w\, v_{ij}(t-1) + c_1\, r_1\, (p_{best,ij} - x_{ij}(t)) + c_2\, r_2\, (g_{best,ij} - x_{ij}(t)) \quad (2)$$

Where i is the index of particle in the swarm, j is the index of position in the particle, t is the iteration number, $v_{ij}(t)$ is the velocity of the i th particle, $x_{ij}(t)$ is the position, w is the positive inertial factor which controls the flying dynamics, and c_1 and c_2 are the acceleration coefficients called learning factor as cognitive and social components respectively. They show how much the particle is directed towards good positions. r_1 and r_2 are two independent uniformly distributed random numbers in the range [0, 1]. p_{best} is the personal best position of a given particle so far. g_{best} is the global best which is the positions among all of the individual best position encountered so far.

The procedures of the PSO algorithm are as follows:

Step 1: Initialize a population of particles. The initial position and the velocity vectors randomly distributed throughout the design space using a uniform distribution for each particle of the swarm which is obtained by Eqs. (3) and (4) (Perez and Behdinan, 2007).

$$x_o = x_{min} + r\,(x_{max} - x_{min}) \quad (3)$$

$$v_o = [\,x_{min} + r\,(x_{max} - x_{min})\,] / \Delta t \quad (4)$$

Where, x_{min} and x_{max} represents lower and upper bounds of the design variables respectively. r is a random number between 0 and 1. Δt is a time step value.

Step 2: For each particle, evaluate the objective function (fitness) values using the design space positions, x_i.

Step 3: Compare particle's fitness evaluation with particle's p_{best}. If current value is better than p_{best} then set p_{best} of that particle and its objective value equal to its current position and objective value.

Step 4: Compare fitness evaluation with the populations overall previous best. If current value is better than , g_{best} then set g_{best} and its value equal to the position and value of the best initial particle.

Step 5: Update the velocity and position of the each particle according to Eqs. (1) and (2).

Step 6: Repeat steps 2-5 until the stop criterion is met. The stopping criterion, termination of the search process, is usually based on the number of iterations assigned or sufficiently good fitness.

In the literature, there are several guidelines for the selection of the key parameters of the PSO algorithm. The velocity, v_i, determines the direction in which a particle needs to move for improving the current position. The particles might fly past good solutions for too high velocity and may not explore sufficiently beyond locally good regions for too low velocity (Eberhart and Shi, 2001). The inertia weight parameter, , which is typically set in the range of [0,1], sensitive to the convergence time (Shi and Eberhart, 1998). It has control over the impact of the previous information of velocities on current velocity of a given particle. A larger value of inertia weight favors a global optimum point while a smaller inertia weight value favors a local optimum point (Kathiravan and Ganguli, 2007). The learning parameters c_1 and c_2 represent acceleration terms that pull each particle toward p_{best} and g_{best} position as a cognitive and social learning respectively.

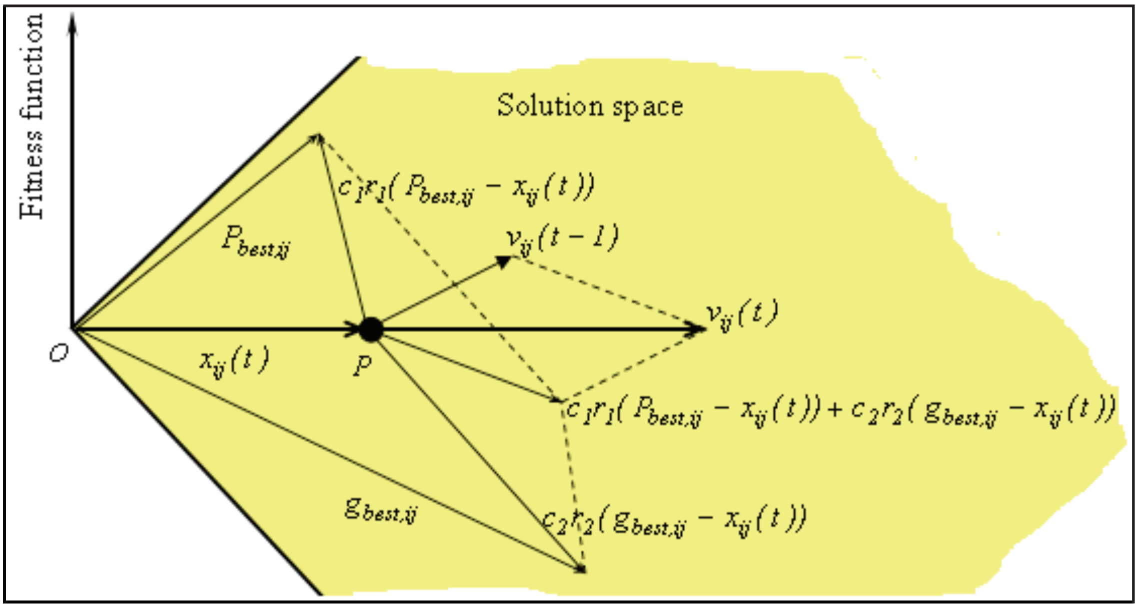

Figure 1. The velocity and position updates.

These parameters usually are defined as constant and are problem dependent. The parameters values in the range [0.5, 2.5] are recommended by Engelbrect (2005) and [1.5, 2.5] by Pulido and Coello (1996). The range for the number of particle (individual) of the swarm is problem dependent, typical 10-200. The more particles, the faster the convergence will be in terms of the number of iterations. The evaluation requires a considerable time when increasing the size of the swarm and less time for decreasing the size of the swarm. It is likely to take longer time to find a solution or even not to find at all when the swarm size is too small. The swarm size should not be longer than necessary. In this study, the parameters for the algorithm are set such as: $c_1=c_2=0.5$, $v_{max}=6.0$, $w=0.5$, particle size $= 100$, and number of iteration $= 10000$.

3. PROBLEM FORMULATION

In this study, the aim is to minimize the volume of the cantilevered beam, see Figure 2. The beam should resist the maximum force, $F_{max}=12000\ N$ and permissible bending stress of the beam material is $\sigma_g=180\ N/mm$. The length, L, of the beam is assumed to be $L=1000\ mm$. The exterior diameter of the beam are assumed to be $D_1=100\ mm$ and $D_2=80\ mm$. For the problem, it is assumed that the interior diameter, x_2, of the beam is to be no less than $40\ mm$. The length, x_1, of the part of the beam can be freely chosen. The problem in hand which was performed in the study by Osyczka (1984) will be used as the reference for the examination and validation of the PSO. The length of the part of the beam, x_1, and interior diameter, x_2 as the design variables are used for finding the minimum volume of the beam. Having restrictions on the design variables and the objective function, constraints are conditions that must be met in the optimum design of the problem. These constraints define the boundaries of the feasible and infeasible design space domain. The constraints considered for the optimum design are the bending strength constraints. The bending strength constraints: one is for the first part of the beam with diameter, D_2 and the other one is for the second part of the beam with the diameter, D_1 .

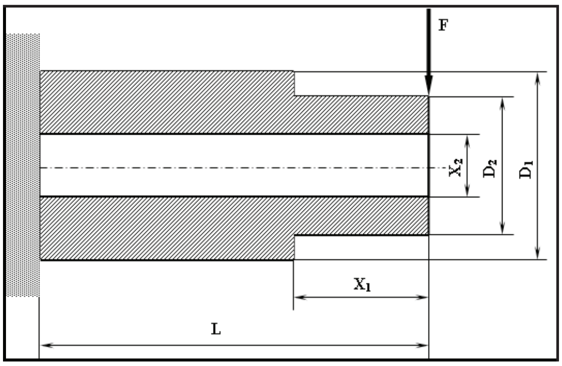

Figure 2. Mathematical model of the beam.

The mathematical expressions of the problem are as follows:

Minimize

$$F_{obj}(x_1,x_2) = \pi[(L-x_1)(D_1^2-x_2^2)+x_1(D_2^2-x_2^2)]/4 \quad (5)$$

Subject to

$$g_1 = [F_{max}\ x_1\ 32\ D_2]/[\pi(D_2^4-x_2^4)] \le \sigma_g \quad (6)$$

$$g_2 = [F_{max}\ L\ 32\ D_1]/[\pi(D_1^4-x_2^4)] \le \sigma_g \quad (7)$$

$$0 \le x_1 \le 800 \quad (8)$$

$$40 \le x_2 \le 75.2 \quad (9)$$

Where, g_1 and g_2 are the bending strength constraints.

The PSO is an unconstrained optimization procedure. Therefore, the objective functions should be transformed into an unconstrained problem by setting an augmented objective function incorporating any violated constraint as penalty function. The most common method to deal with constrained search space is the use of penalty function due to ease implementation. Fitness function, FF, is a summation of the objective function and constraints functions weighted by penalties coefficients. When solutions are feasible, the value of penalty function is zero. When the solutions are infeasible, the value of penalty function is not zero. In case of any violation of a constraint boundary, the fitness of corresponding solution is penalized by penalty function, and thus kept within feasible regions of the design space by increasing the value of the objective function. A unique static penalty function developed by Homaifar v.d., (1994) is used with multiple violation levels set for each constraint in order to maintain a feasible solution. Each constraint is defined by the relative degree of constraint penalty coefficient. The penalty coefficients, r_j, for the *j*-th constraints have to be judiciously selected.

$$FF = \begin{cases} \text{if } (x_1,x_2) \text{ are feasible} \\ \quad F_{obj}(x_1,x_2) \\ \text{otherwise} \\ F_{obj}(x_1,x_2) + \sum_{j=1}^{NC} r_j(\max[0, g_j])^2 \end{cases} \quad (10)$$

Where is number of constraints.

4. RESULTS

Figure 3 shows the plot of normalized objective function value in each iteration as optimization proceeds. From the plot can be seen that the selected parameter set has converged to a stable solution with similar values after generation 48. Figure 4 shows the solution space for the objective function, the beam volume, versus the design variables. It can be seen how the beam volume varies for different design variables combination by visualizing the design space. Also from the plot it can be seen that the optimal point which satisfies the inequality constraints is marked on the plot. The plots in Figure 5 and Figure 6 give the inequality constraints versus the design variables.

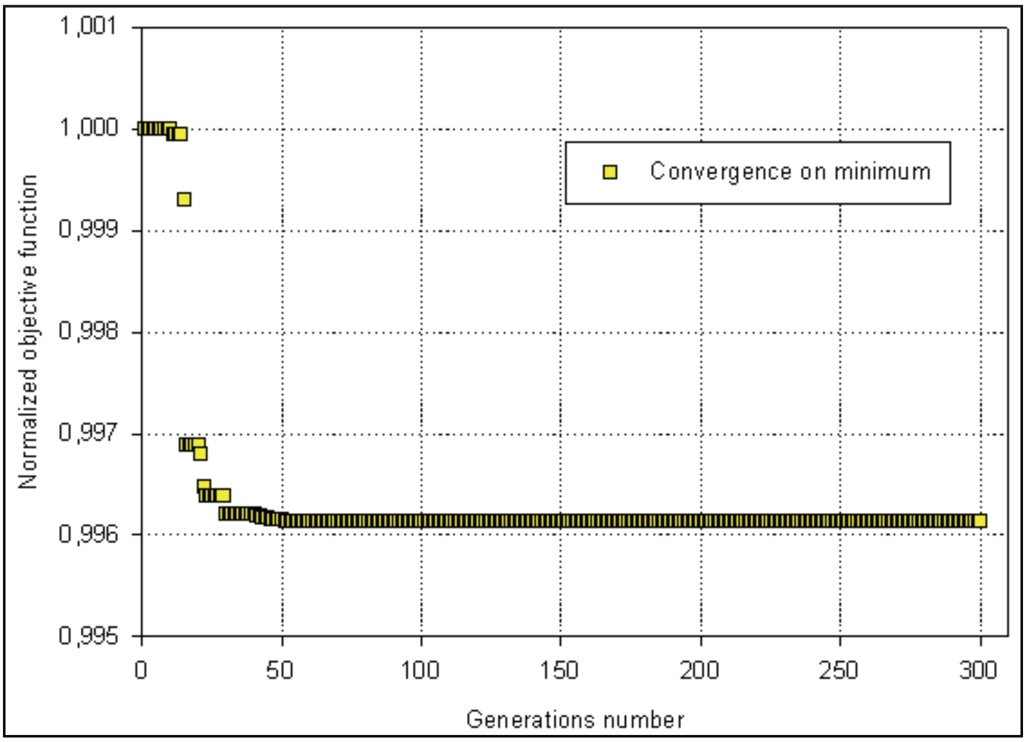

Figure 3. Convergence process of the PSO for best results of objective function.

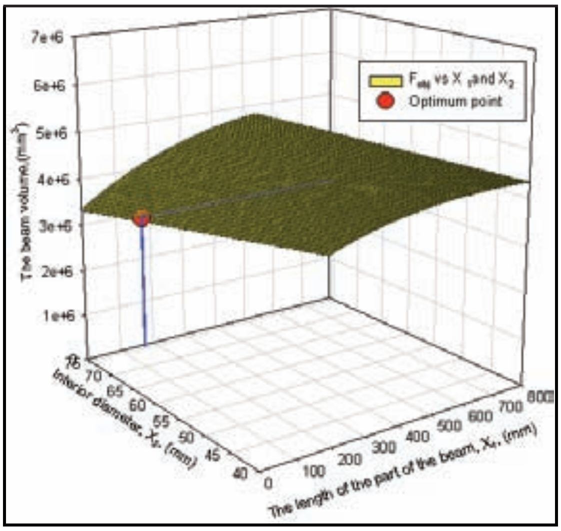

Figure 4. The objective function versus design variables with the optimum point.

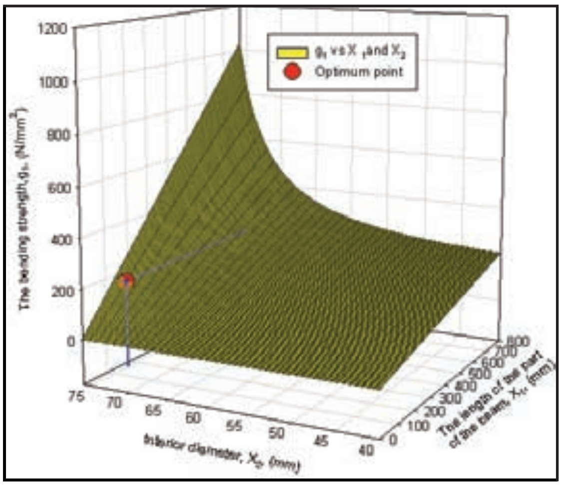

Figure 5. The design variables versus inequality constraint, with the optimum point.

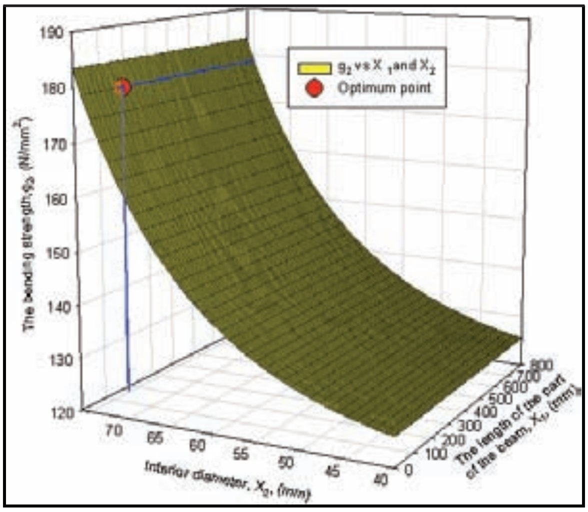

Figure 6. The design variables versus inequality constraint, g_2, with the optimum point.

The plots give the constraints limits which divide the design space in two regions; as feasible design region in which all design constraints are satisfied and infeasible design region in which at least one design constraint is violated. Table 1 shows a comparison of the best overall solution found for the beam volume by the MP and the PSO. As can be seen, the MP gives good approximation to the global optimum but not the exact solution. So, better convergence reliability on global point is obtained with the PSO. Using the MP, the volume of 2961000 mm3 was found with the length of the part of the beam, 159.1 mm, and interior diameter, 75.2 mm, in the study by Osyczka (1984) while the beam volume of 2945345 mm3 was found with the length of the part of the beam, 165.2 mm, and interior diameter, 75.2 mm, by the PSO. The limit of acceptable maximum interior diameter is obtained by the MP and the PSO but not for the length of the part of the beam. For the length of the part of the beam, the PSO result is superior to that from the MP. It can be seen in Table 1, constraints that ensure the conditions of bending strength are satisfied in both methods. Although the MP is a general purpose optimizing tool that can find the best combination of the design variables values which satisfy the constraints placed on properties of the design problem, the MP has experienced difficulties in finding the optimum beam volume compare to the PSO. The problem in hand is carried out for the best combination of the design variables to find the global optimum with no limits on the execution time. The algorithms are repeated until no search direction can be found that will improve the objective function without violating the constraints. Thus, the algorithms are performed for complete search to get the best possible solution of the design. So, the comparison mainly focuses on the empirical solution not the computing time.

Table 1. Comparison of the best overall solution found for the minimum volume of the beam by the MP and the PSO.

	MP	PSO
The length of the part of the beam, x_1, mm	159.1	165.23
Interior diameter, x_2, mm	75.2	75.2
The bending strength constraint, g_1, N/mm	17.32	17.99
The bending strength constraint, g_2, N/mm	17.96	17.96
The beam volume, mm^3	2961000	2945345

5. CONCLUSION

This study employs a nature motivated robust and efficient algorithm, the Particle Swarm Optimization (PSO), to design minimum volume for the given cantilevered beam subject to constraints. It has been shown that the PSO can provide a challenging alternative methodology to the Mathematical Programming (MP) and provide an ability to find global optimum point for mechanical engineering design optimization problems. The PSO is more efficient than the MP when it comes to problems that have numerous locally optimum solutions. In most cases, the MP finds a local optimum that is closest to the starting point. The MP does not offer a guarantee of global convergence. In this study, it was observed that the PSO has been able to find a better solution than the MP. The major disadvantage of the PSO is computation intensive in terms of iterations number. The PSO application in mechanical engineering design optimization is a new area. The results obtained are encouraging.

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