ORIGINAL RESEARCH (AR) DOI: 10.4274/jems.2022.24392

Journal of ETA Maritime Science 2022;10(3):177-184

A Multi-objective Optimization Model for Determining the Performance of a Sailboat

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Abstract

The current research is focused on determining the optimum true wind angle (TWA or β_{TW}). A model is developed to calculate the wind angle at which a sailboat's optimum performance can be achieved. First, the equations for the hull speed, velocity made good (VMG), and the heel angle are determined through the regression analysis by using the data produced by the velocity prediction program analysis. The equations are written for true wind velocities (V_{TW}), with the independent variable being TWA. Later, a multiobjective optimization model is developed, and the wind angles providing the maximum benefit at the respective wind velocities are determined. The goal of the model is to maximize the hull speed and VMG while minimizing the heel angle. The simulated annealing algorithm is employed. Consequently, TWAs providing the optimum performance of a specific sailboat at various wind velocities are calculated.

Keywords: Sailboat, VPP, Performance determination, Optimization, Simulated annealing

1. Introduction

Throughout history, societies have given importance to maritime transport to increase their commercial activities and become richer and more powerful. In this context, freight and passenger transportation between long distances using sailing ships has gained much importance. Owing to technological developments, the uses of sailing ships have turned more toward marine tourism and sporting activities. Motorized or nonmotorized sailboats are used in sports activities to determine the performance of athletes and sailboat. Consequent to economic developments, sailboats with a comfortable interior and a high performance, which can be safe and fast in all weather conditions, are produced.

As in other naval vessels, many scientific studies are conducted on the optimization of the sailboats. These studies are mainly concerned with finding the best route by maximizing the boat speed while minimizing voyage time, as well as determining the optimal shapes and sizes of the boat and the rig.

In this context, if we look at the route optimization problems on sailboats, Wiersma [1] optimized the thrust force contributing to the yacht speed under certain constraints on the lateral force and heeling moment. Day [2] used the computational aerodynamic and hydrodynamic efficiency prediction synthesis to develop methods for estimating the lifting distribution for maximal hull speed. Sugimoto [3] suggested a method for optimum sail strategy, which also performed sail optimization for the maximum yacht speed, was useful in improving the optimum sail design and controlling the optimal sail strategy. Philpott and Mason [4] devised a technique for estimating the minimum-time routes in an uncertain weather. Mairs [5] investigated the flow regimes of two sails experimentally and numerically at different wind angles in his study in which he created an aerodynamic-structural model of windless yacht sails to predict the sail forces. In a separate article, Philpott [6] addressed the use of stochastic optimization methodologies in high-performance yacht racing. Ferguson and Elinas [7] conducted studies on how to reliably determine the best

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Received: 01.02.2022
Accepted: 24.08.2022

To cite this article: M. Kafalı, and E. Aksu, "A Multi-objective Optimization Model for Determining the Performance of a Sailboat." *Journal of ETA Maritime Science*, vol. 10(3), pp. 177-184, 2022.

routes for gaining an advantage in coastal sailing races. Xiao et al. [8] reported that despite the changing wind conditions, the extreme seeker, which maximizes hull speed by changing the sail angle, can pursue the optimum speed of the sailboat on different routes. Tagliaferri et al. [9] stated that they could calculate the minimum estimated time required to reach the opposite direction of the wind by presenting a method for solving the stochastic shortest path problem in races. Dalang et al. [10] proposed mathematical sailboat racing methods that used the statistical analysis of wind disturbance and were useful for the stochastic optimization methods. Tagliaferri and Viola [11] presented an optimal strategy in sailboat racing to complete the race in the minimum expected time. Ferretti and Festa [12] showed that the hybrid control method can be used to plan sailboat routes in the shortest possible time. Sarı and Aydın [13] investigations on the sailing yachts aimed to establish the ideal sailing parameter dimensions and forecast performance values for the given sail and boat. Kemali et al. [14] utilized computational fluid dynamics to examine the impact of the leading-edge tubercles of the wing-type sails used in the 2013 America's Cup.

In this study, a multiobjective optimization problem is defined to maximize the speed of a boat over ground and the VMG, and to minimize the angle of inclination. The true wind angles (TWAs) exhibiting the best performance at different wind velocities have been determined.

2. Materials and Methods

A hierarchic model combining VPP and multi objective optimization is built in this study (Figure 1).

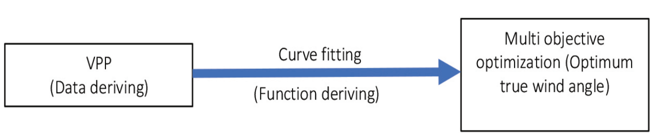

Figure 1. *The model used to determine optimum TWAs*

Sailing boat speed can be estimated using VPP, based on the balance of forces and moments acting on a sailboat. In the first step, a sailboat is analyzed to generate data using the Bentley Systems' VPP software (www.bentley.com). In the second step, a regression analysis is performed to find the fitted equation for each function based on the data generated in the previous step. The third step involves obtaining optimum TWAs according to the defined objective functions. Information on all steps is given in the following subsections.

2.1. Description of the Sailboat

In various sizes and sail configurations, sailboats that move by using wind power are produced. The sailboat in the application is a Bentley Systems VPP software sample design. The sailboat (Figure 2) consists of a mainsail that catches a large part of the wind and provides the required propulsion, a headsail that increases the air flow by steering the wind to the front of the mainsail, a spinnaker, which is a downwind sail that balloons to increase the boat's efficiency, and a keel for hydrostatic resistance that allows the sailboat to navigate upwind and provides some stability by lowering the center of the gravity of the boat. The dimensions and the sailing equipment of the sailboat are listed in Table 1.

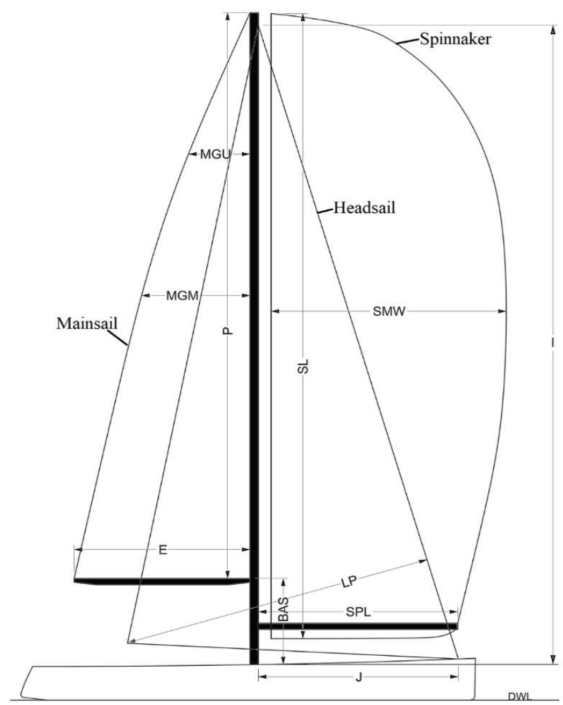

Figure 2. *Sailboat with three sails [15]*

2.2. Deriving Data for Optimization

To obtain equations for hull speed, heel angle, and VMG, data generated by using Bentley Systems VPP software. MAXSURF VPP is a widely used software for predicting the sailboat's performance. By resolving the lift and drag equations for the hull and the rig, it determines the equilibrium velocity and the heel angle. The sample design and measurement data in its library is used. This study deals with true wind velocity and angle as the input, and with hull speed, heel angle, and VMG as the output. In this regard, six true wind velocities are determined to reflect low, medium, and high wind force. The parameters used in the optimization are shown in Figure 3.

***Table 1.** Dimensions of hull and the sail [15]*

HULL				MAINSAIL	
Length W.L.	10.36 m			The luff length of the mainsail (P)	14.783 m
Beam W.L.	2.508 m			The foot length of the mainsail (E)	4.203 m
Draft	2.44 m			The upper girth length of the mainsail (MGU)	1.554 m
Displaced volume	4.013 m^3			The middle girth length of the mainsail (MGM)	2.743 m
Block coeff.	0.059			Length of the lower mainsail luff band (BAS)	2.102 m
Prismatic coeff.	0.484			**HEADSAIL**	
Max. sec. area coeff.	0.129			The distance measured between the sheer line and the top of the foretriangle (I)	16.605 m
Waterplane area coeff.	0.702			Distance between the headstay base and front of the mast (J)	4.849 m
KEEL DIMENSIONS				Perpendicular distance from the headsail clue to the luff (LP)	7.602 m
	Length	Beam	Depth	**SPINNAKER**	
Bulb: NACA 65-015	1.6 m	0.286 m	0.25 m	Pole length of the spinnaker (SPL)	4.871 m
Bulb keel: NACA 64-010	0.746 m	0.068 m	2.2 m	Luff length of the spinnaker (SL)	16.002 m
Ballast ratio: 0.45				Maximum width of the spinnaker (SMW)	8.778 m

For each true wind velocity, upwind and downwind situations are calculated at certain TWAs (Table 2). As an example, the calculated data in the upwind sail for a 6-knot (kn) true wind velocity is given in Table 3. Here, VMG represents the speed component in the reverse wind direction. When the wind is blowing at an angle of over 90 degrees and is coming from behind the rig, VPP yields negative VMG values. The Reef factor is constantly set to 1 in the calculations. Figure 4 shows the polar plots for the VPP results.

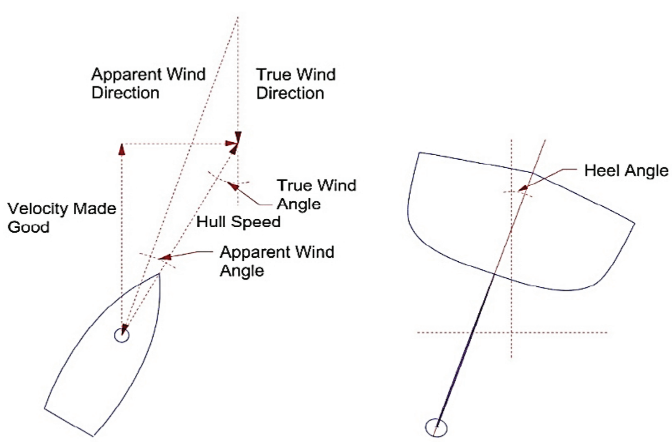

***Figure 3.** Concepts employed in the optimization model*

2.3. Optimization Model

The optimization model is concentrating to determine the optimum TWAs. In this regard, two main scenarios are considered: downwind and upwind. Two functions are included in the downwind case, i.e., the "hull speed" and "heel angle." Both depend on TWA at a certain true wind velocity. Conversely, in the upwind case, one more function is added, i.e., VMG. Thus, there are three functions in the upwind situation: "hull speed," "heel angle," and "VMG." These functions vary with TWA at a given true wind velocity, as in a downwind situation.

Regression analysis is used to derive the hull speed, heel angle, and VMG functions. These functions are used in the multi objective optimization. In both cases, six different true wind velocities are considered. VMG is not included in the

***Table 2.** The input, output, true wind velocity, and TWA values in both situations*

Sail course	Input		Output
Upwind	True wind velocity (V_{TW}) (kn)	6, 10, 14, 18, 22, 26	Hull speed Heel angle VMG
	TWA (β_{TW}) (deg)	35, 38, 41, 44, 47, 50, 53, 56, 59, 62, 65, 68, 71, 74, 77, 80, 83, 86, 89, 92, 95, 98, 101, 104, 107, 110	
Downwind	True wind velocity (V_{TW}) (kn)	6, 10, 14, 18, 22, 26	Hull speed Heel angle
	TWA (β_{TW}) (deg)	80, 83, 86, 89, 92, 95, 98, 101, 104, 107, 110, 113, 116, 120, 123, 126, 129, 132, 135, 138, 141, 144, 147, 150, 153, 156, 159, 162, 165, 168, 171, 174, 177, 180	

Table 3. *VPP results for a 6-kn true wind velocity in the upwind sail*

β_{TW} deg	35	38	41	44	47	50	53	56	59	62	65	68	71
Hull speed kn	4.6	4.93	5.23	5.51	5.76	6	6.21	6.39	6.54	6.68	6.79	6.89	6.98
VMG kn	3.76	3.89	3.95	3.96	3.93	3.86	3.74	3.57	3.37	3.13	2.87	2.58	2.27
Heel angle deg	13.26	14.85	16.08	17.01	17.74	18.31	18.76	19.12	19.43	19.69	19.9	20.06	20.18
β_{TW}deg	**74**	**77**	**80**	**83**	**86**	**89**	**92**	**95**	**98**	**101**	**104**	**107**	**110**
Hull speed kn	7.05	7.12	7.18	7.22	7.24	7.24	7.23	7.2	7.16	7.1	7.03	6.93	6.81
VMG kn	1.94	1.6	1.25	0.88	0.5	0.13	−0.25	−0.63	−1	−1.36	−1.7	−2.03	−2.33
Heel angle deg	20.27	20.33	19.34	17.36	15.51	13.77	12.12	10.54	9.02	7.55	6.12	5.13	4.49

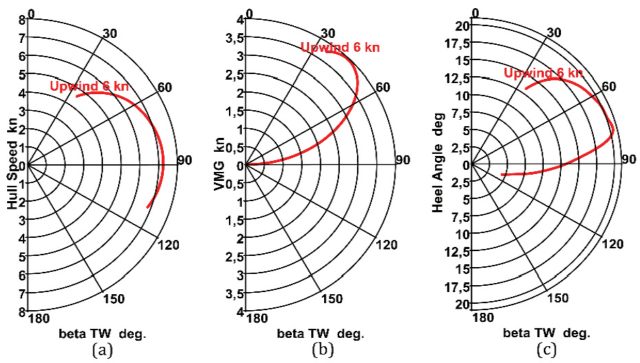

Figure 4. *Polar plots for the VPP results (a) Hull speed; (b) VMG; (c) Heel angle*

downwind situation as it takes negative values for multiple TWAs and is therefore, not included in the optimization model (Table 4).

Table 4. *Parameters for the upwind and downwind cases*

Upwind situation		Downwind situation	
Function	Independent variable	Function	Independent variable
Hull speed Heel angle VMG	TWA (β_{TW})	Hull speed Heel angle	TWA (β_{TW})

Following the formulation of functions, a multi objective optimization model is constructed for both situations. There are two objective functions in the downwind situation and three objective functions in the upwind situation. By using the weighted-sum method, both models are converted into a single objective optimization, and the simulated annealing (SA) algorithm is used to determine the optimal solutions. SA has been introduced by Kirkpatrick et al. [16] and Černý [17]. This algorithm is a probabilistic search technique and was developed inspired by the annealing process in metalworking. A heuristic mechanism is specified in the SA algorithm to avoid sticking to the local optimum. This heuristic mechanism works by performing a random search that accepts the neighboring solutions improving the objective function, but also some of those not improving the objective function [18].

Probability of acceptance is calculated using equation 1 where ΔE is the difference between the calculated value of the objective function according to the neighbor solution and its current value [19].

$$P(\Delta E, T) = e^{-\frac{f(s')-f(s)}{T}} \tag{1}$$

The acceptance probability is compared to a random number generated between 0 and 1. If $P(\Delta E, T)$ is greater than the generated random number, the neighboring solution that does not improve the objective function is accepted [20]. The occasional acceptance of the neighboring solutions, worsening the objective function value, frees the SA algorithm from being stuck with the local optimum. Bad solutions are more likely to be accepted at higher temperatures, i.e., early in the search [19].

2.3.1. Multi Objective Optimization

In the weighted sum method, objective functions are weighted and aggregated. For the downwind (2) and upwind (3) situations, optimization models are given below:

$$\begin{aligned} min\left(\bar{F}_{dw}\right) &= w_1\bar{f}_1(x) + w_2\bar{f}_2(x) \\ w_1 + w_2 &= 1 \end{aligned} \tag{2}$$

$$\begin{aligned} min\left(\bar{F}_{uw}\right) &= w_1\bar{f}_1(x) + w_2\bar{f}_2(x) + w_3\bar{f}_3(x) \\ w_1 + w_2 + w_3 &= 1 \end{aligned} \tag{3}$$

where, the subscripts *dw* and *uw* stand for the downwind and upwind situations, respectively. $\bar{f}_i$ denotes the normalized function. $\bar{f}_i(x)$ is calculated by equation 4 [21]. w_i represents the weight of the objective function. Since w_i is so important in the weighted sum method, several w_i combinations are considered in this study. More information on the w_i values can be gathered from the findings and discussions section. Functions depend on the TWA and velocity. However, as mentioned before, six true wind velocities are examined separately so that at a certain true wind velocity, each function depends on TWA only. Hence, *x* in the equations above stands for TWA.

$$\bar{f}_i(x) = \frac{f_i(x) - f_{i,min}(x)}{f_{i,max}(x) - f_{i,min}(x)} \tag{4}$$

In the equations (2), (3), and (4), the indices stand for a function mentioned in Table 5.

***Table 5.** Subscripts and their meanings*

Subscript (i)	Meaning
1	Hull speed
2	Heel angle
3	VMG

As seen in the optimization model, the aggregated function is minimized. However, the hull speed and VMG must be maximized. To accomplish that goal, the hull speed function is converted to its inverse form. Let us assume that the hull speed function is $f'_1(x)$, then $f_1(x) = \frac{1}{f'_1(x)}$. In the upwind case, VMG takes a negative value for certain TWAs; thus, the same principle does not apply here. To address this issue, a value greater than the maximum of VMG data is identified, and the difference between the two is computed. Say, VMG function is $f'_3(x)$, then $f_3(x) = M - f'_3(x)$. Here M stands for the big value.

3. Findings and Discussions

Determination of the weights (w_i) of the objective functions is quite crucial in the weighted sum method. Therefore, in this study, the model is analyzed for several weight combinations. The weights are raised or reduced at the intervals of 0.25 for convenience. The optimization model gives a different solution for each weight combination. These results represent a variety of cases that could be optimal for various scenarios. The detailed findings for both the upwind and downwind sails are given and discussed in the following subsections.

3.1. Downwind Course

It is possible to show all weight combinations in a single graph for this condition. The results obtained for the downwind condition are reflected in Figure 5. w_1 and w_2 represent the weight of the optimization parameters. $w_1 = 1$ and $w_2 = 0$ mean the importance of the hull speed equals 1, and that of the heel angle equals 0 in the optimization. The term "importance" refers to the impact of the related factor on the optimization. Accordingly, the "$w_1 = 1$; $w_2 = 0$" and

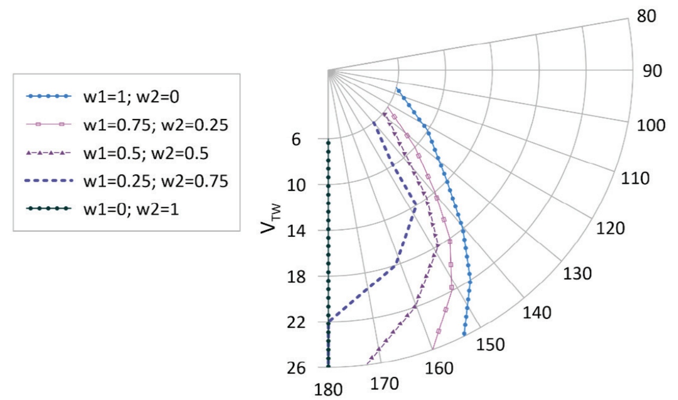

***Figure 5.** Optimization results for optimum β_{TW} at the downwind course*

"$w_1 = 0$; $w_2 = 1$" weight combinations represent the single objective cases for the maximum hull speed and the minimum heel angle, respectively. As expected, the optimal β_{TW} (TWA) for the minimum heel angle is high and equal to 180 degrees. Meanwhile, the optimal β_{TW} varies according to the V_{TW} (true wind velocity) for the maximum hull speed. The optimum angle is also relatively low at low V_{TW}s and stays below 135 degrees. The optimal β_{TW} shifts toward 150 degrees as the V_{TW} increases.

Other weight combinations depict the multiobjective scenarios. When we look closely, we see that when the importance of the heel angle rises, so does the value of optimum β_{TW}. This increase is more restricted at lower V_{TW} s and more apparent at higher V_{TW}s such that when V_{TW} is over 22 knots, β_{TW} equals 180 degrees. Generally, from low to medium V_{TW}s, the optimum β_{TW} ranges from 120 to 150 degrees. Conversely, from medium to high V_{TW}s, the optimum β_{TW} ranges from 140 to over 170 degrees. Apparently, a set of intersection angles exists for both states. In addition, at the extreme ends of V_{TW}, optimum β_{TW} comes closer to single objective optimum solutions. Notably, no optimum β_{TW} between 110 and over 140 degrees has been determined from medium to high V_{TW}s. Similarly, no optimum β_{TW} between 150 and 180 degrees has been detected from low to medium V_{TW}s, except the single objective condition of the heel angle. Generally, it can be claimed that if the heel angle is much more important, β_{TW} should be high. Conversely, if the hull speed is much more important, β_{TW} should be relatively low.

3.2. Upwind Course

In this case, three objective functions are considered. The results of multiple weight combinations are presented in separate graphics. w_1, w_2, and w_3 are the parameter weights. Accordingly, w_1 stands for the weight of the hull speed, w_2 stands for the weight of the heel angle, and w_3 stands for the weight of the VMG in the optimization. In Figure 6, the curve belongs to the "$w_1 = 0$; $w_2 = 1$; $w_3 = 0$" combination, intersects the 10-knot radius at the two points because of a sudden change in the optimum β_{TW} and the structure of the polar graph. However, the first intersection must be considered since only one optimum β_{TW} value has been calculated for each V_{TW}. The same issue exists in some subsequent cases, and the same explanation applies to them.

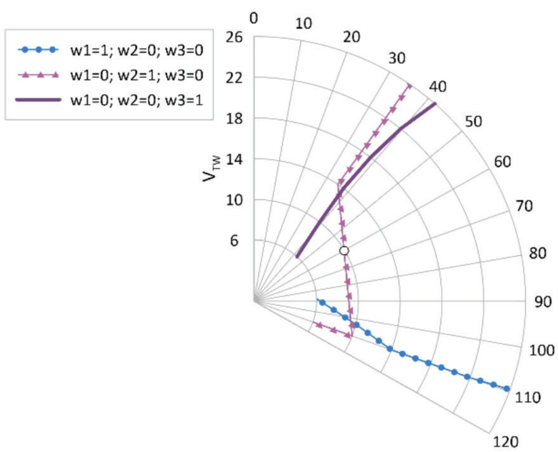

Figure 6. *Single objective optimization results for optimum β_{TW} at the upwind course*

A close examination of Figure 6 reveals that the optimum β_{TW} changes between 90 and 110 degrees only for the maximum hull speed. At the lower V_{TW}s, it is close to 90 degrees, while being close to 110 degrees at medium and high V_{TW}s. When only the heel angle is considered, optimum β_{TW} varies in an interesting way. While the optimum β_{TW} is 110 degrees at lower V_{TW}s, it suddenly changes to 35 degrees at the medium and high V_{TW}s. Because of the sudden change, the curve in Figure 6 intersects the 10-knot radius at the two points. However, only the first intersection is valid. The other intersection occurs because of the polar graph's structure. When VMG is considered solely, the optimum β_{TW} has a consistent behavior and varies around 40 degrees at all V_{TW}s.

Figure 7 represents the biobjective cases. The first graph (Figure 7a) exhibits the case where the weight of the hull speed is zero. Therefore, it reflects the change in the optimum β_{TW} for different weight combinations of the heel angle and VMG. Accordingly, when the importance of the heel angle is high, optimum β_{TW} exhibits similar attitude with a single objective case that only the heel angle has been considered. Hence, at lower V_{TW}s, the optimum β_{TW} is 110 degrees. At the medium and high V_{TW}s, the optimum β_{TW} is 35 degrees. When the importance of the heel angle and VMG is equal, the behavior of the change of the optimum β_{TW} is similar (except 6 knots) to that in the single objective case where only the heel angle is considered. When the case wherein the weight of the VMG is much more important than the heel angle is considered, the optimum β_{TW} displays stability and varies around 35 degrees at all V_{TW}s.

The second graph (Figure 7b) exhibits the case where the weight of the heel angle is zero. Therefore, it reflects the change in the optimum β_{TW} for different weight combinations of the hull speed and VMG. Accordingly, when the importance of the hull speed is high, the optimum β_{TW} changes between 70 and 100 degrees at lower V_{TW}s. At the medium and high V_{TW}s, the optimum β_{TW} exhibits a more stable character, and the optimum β_{TW} changes between 85 and 90 degrees approximately. When the importance of the hull speed and VMG is equal, the optimum β_{TW} displays a stable form and varies around 60 degrees at all V_{TW}s. Considering that VMG is significantly more important than the hull speed, the optimum β_{TW} displays a stable character and varies around 50 degrees at all V_{TW}s.

The third graph (Figure 7c) signifies that the weight of VMG is zero. Therefore, it reflects the change in the optimum β_{TW} for different weight combinations of the hull speed and the heel angle. Accordingly, if the importance of the hull speed is higher than the heel angle, optimum β_{TW} changes between 100 and 110 degrees. At medium V_{TW}s, the optimum β_{TW} is close to 100 degrees, while it changes around 110 degrees at low and high V_{TW}s. When the importance of the hull speed and the heel angle is equal, the optimum β_{TW} changes a lot according to the V_{TW}. Hence, at low V_{TW}s, the optimum β_{TW} is steady and changes around 110 degrees. Meanwhile, at medium V_{TW}s, it changes between 40 and 85 degrees approximately. Therefore, under these circumstances, it exhibits a more unstable character. At higher V_{TW}s, it differs and draws a more stable character and changes around 80 degrees. When the weight of the heel angle is considerably more than the hull speed, the optimum β_{TW} displays a similar character with a single objective case that only heel angle has considered. Thus, while the optimum β_{TW} is 110 degrees at lower V_{TW}s, it suddenly changes to 35 degrees at medium and high V_{TW}s.

Figure 8 illustrates the triobjective cases. The conditions seen in this figure include all the objective functions. The red dotted curve indicates the condition in which the weight of the hull speed is slightly more than other factors of equal importance. Accordingly, at lower V_{TW}s, the optimum β_{TW} changes between 100 and 110 degrees. At medium V_{TW}s, it differs and draws a more unstable character and changes between 50 and 75 degrees. At high V_{TW}s, the optimum β_{TW} is

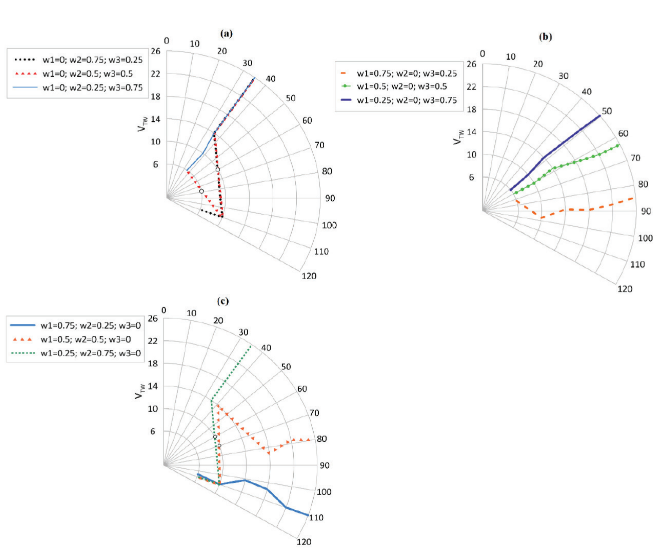

Figure 7. *Biobjective optimization results for optimum β_{TW} at the upwind course*

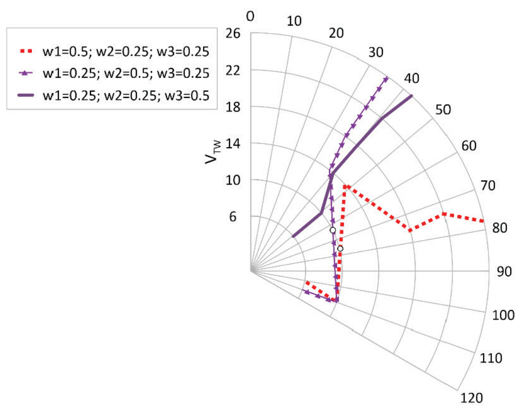

Figure 8. *Triobjective optimization results for optimum β_{TW} at the upwind course*

again more stable and changes around 75 degrees.

The purple triangle curve indicates the condition in which the weight of the heel angle is slightly more than other factors of equal importance. In this condition, the optimum β_{TW} exhibits a similar attitude with the single objective case where only the heel angle is considered. Hence, at lower V_{TW}s the optimum β_{TW} is 110 degrees. At medium and high V_{TW}s, the optimum β_{TW} is around 35 degrees.

The continuous curve indicates the condition in which VMG's weight is slightly more than other factors of equal importance. Accordingly, at lower V_{TW}s, the optimum β_{TW} is 50 degrees. At medium and high V_{TW}s, the optimum β_{TW} exhibits close resemblance with the single objective case where only VMG is considered. Consequently, it fluctuates around 40 degrees.

4. Conclusion

Sailboats are watercrafts that sail the oceans by using wind power. They come in a range of sizes and sail configurations. For sailboats to perform well, the optimum wind angle must be identified. Consequently, a model is constructed to calculate the TWAs at which a sailboat can achieve the best performance. A multiobjective optimization problem is also included in this model. At respective wind velocities, the TWAs that give the greatest benefit are calculated. In this research study, a particular sailboat is investigated. The model considers a variety of different weight combinations. The model's output represents a set of situations that may be suitable for different scenarios. The findings in the downwind and upwind scenarios are different. While the significance of the hull speed increases in the downwind case, the optimum β_{TW} moves from 180 degrees to lower values. Since three objective functions exist in the upwind case, it is a little more complicated. However, in the general terms, when the importance of the hull speed is high, optimum β_{TW} approaches higher values, i.e., toward 110 degrees. When the importance of VMG is high, optimum β_{TW} shifts to lower values, i.e., toward 40 degrees. When the importance of the heel angle is high, optimum β_{TW} varies according to the wind velocity. Thus, it shifts toward higher values, i.e., toward 110 degrees at the higher true wind velocities. In contrast, it shifts toward lower values, i.e., toward 35 degrees at lower true wind velocities.

In the future research, various types of sailboat hulls can be studied and compared. Moreover, different mainsails and jibs can be investigated for sailboat performance in optimization studies. Meanwhile, objective functions can be changed according to the desired goals. Additionally, the objective functions can be modified to meet specific targets. In this case, new outcomes could be appropriate to accomplish the expectations.

Authorship Contributions

Concept design: M. Kafalı, E. Aksu, Data Collection or Processing: M. Kafalı, E. Aksu, Analysis or Interpretation: M. Kafalı, E. Aksu, Literature Review: E. Aksu, Writing, Reviewing and Editing: M. Kafalı.

Funding: The author(s) received no financial support for the research, authorship, and/or publication of this article.

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