AL KHWARIZMI'S CONTRIBUTIONS TO THE SCIENCE OF MATHEMATICS: AL KITAB AL JABR WA'L MUQABALAH

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SUMMARY: Abu Abdullah Ibn Musa Al Khwarizmi (780-847), the first great mathematician of the Islamic world, was also the founder of Algebra. Many books about the philosophy of mathematics and the history of mathematics referred to his studies. Today we have the Arabic writings of Al Khwarizmi who is considered as one of the most important Islam innovators and the first medieval algebraist. This paper is an attempt to show how Al Khwarizmi's studies influenced the philosophy of mathematics and the development of the advanced algebra.

Key Words: Mathematics, algebra.

INTRODUCTION

Al Khwarizmi worked in the ninth century under the patronage of the Caliph Al Ma'mun in Baghdad. Al Khwarizmi became a member of *Dar-ul Hikme* which means the house of wisdom, a kind of academy of scientists founded in Baghdad most probably by Caliph Kharun Rhasid. But it owes its preeminence to Al Ma'mun, a great patron of learning and scientific investigation. At the time of Al Ma'mun, not only the science of mathematics but also other sciences lived a golden age.

Early in his career, Al Khwarizmi went to Afghanistan and then to India, he met Indian scholars. After coming back to Baghdad, he introduced Hindu mathematics and astronomy. At that time, he wrote an astronomical table which is known in Arabic as *Sindhind*. In this period, he taught addition, subtraction, multiplication, and division for traders, survey officers, and finance officers at *Dar-ul Hikme*. He taught some particular calculations that were important for *Cadis* (Islamic lawyers) to implement the law of heritage in Islam.

While teaching at *Dar-ul Hikme*, Al Khwarizmi published his most popular book *Al Kitab Al Jabr Wa'al Muqabelah* which was written in Arabic in 830. This book made him famous in the east and west. The influence of this book lasted for centuries. If we consider that *Al Kitab Al Jabr Wa'al Muqabelah* was translated and published in London in 1831 and in New York in 1945, we can say that Al Khwarizmi's work strongly influenced the west for a long time. His work became a base of the study of algebra in Renaissance. It is safe to say that big mathematicians such as Leonardo Fibonacci (1175-1230), Alberd (1196-1280), and Roger Bacon (1214-1294) used Al Khwarizmi's algorithms and solutions in their studies. This influence clearly appears in *Practica Geometria* written by Leonardo Fibonacci.

In the philosophy of mathematics, positivists claimed that mathematical knowledge is objective and can cover the external reality. Having argued against the searching absolute truth, today many mathematicians who take a constructivist position see mathematical knowledge as an

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individual construction. So, it is subjective knowledge and there is no authority to evaluate the match between individual construction and ultimate reality. This implies that mathematical knowledge constructed by human mind is limited to cover the ultimate reality. More than eleven hundred years ago Al Khwarizmi addressed his philosophy about the absolute truth in mathematics by giving an example for the calculation of the circumference of a circle. His method is very close to the modern calculation. This method was translated by Rosen (4) in his book:

"This is an approximation, not the exact truth itself, nobody can ascertain the exact truth of this and find the real circumference, except omniscient, for the line is not straight so that its exact length might be found... The best method here given is that you multiply the diameter by three and one seventh; for (p.200)."

It means that God can know his creation because he created it, we however, can only know what we ourselves have created. Although modern mathematics does not focus on any ontological issue and consideration of God's creation, a Dutch mathematicians Brower thousand years after, presented a counter-example to the law of trichotomy by using the same example used by Al Khwarizmi. Brower's example illustrates the time dependent and subjective character of mathematical truth (1).

Al Kitab Al Jabr Wa'al Muqabelah has been translated into English as *The Book of Restoration and Balancing.* In his algebra, Al Khwarizmi did not use symbols but expressed everything in words: For the unknown quantity he employs the word shay (thing or something). For the second power of the quantity he employs the word *mal* (wealthy). For the unit he uses *dirham.* The idea of root was explained as a line x^1. There is a duality in the measuring of *jadh* (root) because an area corresponds to the first power x. Thus *jadh* as the side of a square (x) multiplied by the square unit x^1.

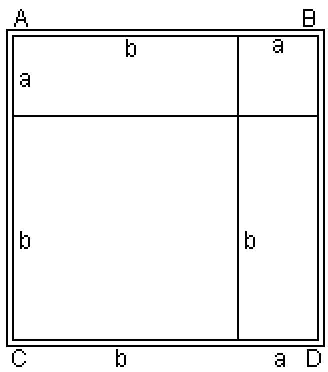
Al Kitab Al Jabr Wa'al Muqabelah includes forty different problems. The book consists of one preference, appendix and five main chapters. These chapters can be summarized as following:

The first chapter

This chapter has two parts. The first part includes six different equation types (linear and quadratic). Al Khwarizmi gives geometric solution for them. In the history of mathematics this solution method appeared first time in this book. These equations were classified by Göker (2) into six standard forms that are given with modern notations:

1) $ax^2=bx$
2) $ax^2=b$
3) $ax=b$
4) $ax^2+bx=c$
5) $ax^2+c=bx$
6) $ax^2=bx+c$, where a, b and c are positive integers.

The second part of this chapter includes the method of multiplication (a+b).(a+ b) and (a-b).(a-b). An example of this method has been given by Al Khwarizmi; he showed the multiplication of (a+b).(a+ b) as the area of the square ABCD constructed by (a+ b):

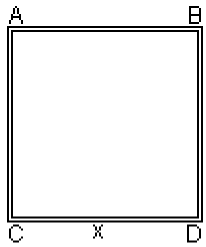


Thus, the total area of the (a + b) square is $b^2 + a^2 + 2ab$.

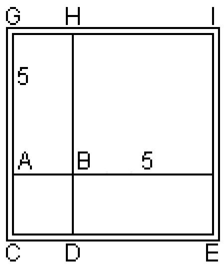
The second chapter

This chapter is very original in the history of mathematics dealing with practical menstruation. Al Khwarizmi applied a geometric model to solve quadratic equations, he called this application the square and rectangle method. He solved the equation of $x^2+10x=39$ by using this method.

The same solution with modern notations can be found in the chapter on Al Khwarizmi in Abu Kamil (3). In this solution, Al Khwarizmi draws a square ABCD of side x.



He adds 5 units to side AC and CD in order to construct the square DGIE:



Thus, the total area of the square DGIE is $(x+5)^2$, it is also equal to $x^2+10x+25$. We know that $x^2+10x=39$. If one puts this value into the equation of $x^2+10x+25$, then the area of the square is equal to 39+25=64. Therefore, $(x+5)^2=64$, (x+5)=+8 or -8, then x=3 or x=-13.

This solution is the first example of Analytic Geometry. We cannot see it in early Greek, Egyptian, and Indian mathematics.

The third chapter

This chapter is generally deals with how to find the result of Binome formulas such as a two term multiplier whose one term is unknown. He also explains how to factor some equations which can be written by means of modern notations:

(x+a).(x+b)
(x+a).(x-b)
(x-a).(x+b)
(x-a). (x-b)

The fourth chapter

In this chapter, Al Khwarizmi gives some solutions for different equation types. His rule by modern notations can be expressed as:

i) $x^2 = ax \qquad \sqrt{x}\sqrt{x} = b\sqrt{x}$

then x = a

ii) $x = b\sqrt{x} \qquad \sqrt{x} = b$

where a, b, and c are positive integers.

iii) $x = c\sqrt{x}$

then $x = c^2$

A numerical example of his solution is given the following:

$$\left(x - 3\sqrt{x}\right) = \sqrt{x}$$
$$x = 4\sqrt{x}$$
$$\sqrt{x}\sqrt{x} = 4\sqrt{x}$$
$$\sqrt{x} = 4$$
$$x = 16$$

The fifth chapter

This chapter includes some problems which can be solved algebraically. Especially, Al Khwarizmi mentioned four arithmetic operations (addition, subtraction, multiplication, and division). An example of his original algebraic solution is expressed with modern notations:

The sum of two numbers is 10, and the subtraction of squares of these numbers is 40.

$x+y=10$
$x^2-y^2=40$
Let x=5+z and y=5-z in modern notations.
$(5+z)^2-(5-z)^2=40$

$25+10z+z^2+z-25-z^2=40$

$20z=40$

$z=2$ then $x=5+2=7$, and $y=5-2=3$.

Appendix

This part includes some special topics which are necessary for government jobs. These topics can be listed as:

1. The digging of canals
2. The measuring of lands
3. The using of Hindu number system
4. Geometrical and arithmetical computations for inheritance law in Qur'an.

In this paper, I attempted to illustrate Al Khwarizmi's profound effect on the philosophy of mathematics and the development of algebra by describing his book with modern notations. Today one can see Al Khwarizmi's influence not only in the philosophy and scientific history but also in mathematical texts that were written in the centuries after him. This overt influence can be easily seen on everyone who followed him such as Abu Kamil, Omar Khayyam, Abul Wafa, Fibonacci, and Descartes. Thus, Al Khwarizmi's work provided a scientific resource for mathematicians to improve advanced algebra. Over the centuries writers on basic algebra have borrowed his format, his terminology, and his classification of the types of linear and quadratic equations that he did in his original algebra.

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