# A NEW TECHNIQUE (HOLOTRANSFORMATION) FOR EVALUATION OF A FORCED CONVECTION EQUATION FROM EXPERIMENTAL DATA*

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*SUMMARY: In the mathematical modelling of engineering studies, when we want to make an evaluation on the experimental data we meet a significant problem that is the elimination of uncertainties in observed data. Its nature and magnitude is associated with both measurement error and our models of the systems of chemical, biological and environmental. In this work, a new technique (Holotransformation) is proposed for evaluation of a forced convection equation from experimental data. It has been demonstrated and compared by an example which is carried out "Proces Heat Transfer", D.Q. Kern, Mc Graw-Hill. P. 56-53 (1950) (6). By the Holotransformation, the problem is reflected from Eucludian space into a non-Eucludian space by a suitable transformation matrix. The solution of problem is always made in non-Eucludian space by the conventional techniques. A back reflection (Holotransformation) gives Euclidian space's solution which is the solution of original problem. The application of Holotransformation on the solution of normal equations of linear least squares problem shows that it gives an ability to find out more proper and simplest mathematical formula for observed data.*

*Key Words: Holotransformation, forced convection equation.*

INTRODUCTION

The Holotransformation is a new transformation which is developed and reported by Author (1,2,4). The Author developed three new approaches for the solution of linear least squares (LLS) problem by using Holotransformation as following;

a) The Artificial Variation Method (AVM)

b) The Equivalent Normal Equations Method (ENEM)

c) Direct Approach Method (DAM)

In this work, the normal equation of test problem has been solved by the Artificial Variation Method (AVM). The normal equations is reflected with respect to the new coordinate system ( in new space) by reflection matrix E which will be defined as $E=(A^T.A)^T$ for $(A^T.A)\ X=A^T.\ b$ normal equation. The reflected normal equation will be $(A^T.A).$ $(A^T.A)^T\ \tilde{X} = A^T.b$ by the application of (AVM).

In the regression analysis fitted model has some residuals, or unexplained variations. Those unexplained variations remains, the model might be uncorrect or there might be noise or other uncertainties in the experimental data. Sometimes it is need this variation to get best model. In this work this artificial variation is making by multiple application of a kind of Holotransformation that gives an ability to find out better model for observed and measurement data without changing type of model (4).

THE PROPOSED ALGORITHM BY HOLOTRANSFORMATION

The proposed algorithm leads to the solution of normal equations in three steps which are given by the theorem mentioned below. Namely;

a) To convert the original normal equation into the auxiliary normal equations.

b) To solve the auxiliary normal equations by using any conventional method.

c) To obtain the final solution by using the result of auxiliary normal equations.

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THEOREM

Let AX=b, a linearly independent system and A matrice will be square or not, it can be solved by using following matrice operations.

a) The coefficient matrice of the original linear system transposes into $A^T$ and $(A.A^T)$ matrice multiplication determines the coefficient matrice of the auxiliary linear system which is always symmetric.

b) To solve the defined $(A.A^T)\tilde{X} = b$ auxiliary linear system by using any conventional method. The (b) vector of the original linear system in the same (b) vector of the auxiliary linear system. The auxiliary linear system is defined by $(A.A^T)\tilde{X} = b$.

c) The solution set $\tilde{X}$ of the auxiliary linear system transposes into $\tilde{X}^T$ and $(\tilde{X}^T.A)$ vector matrice multiplica determines the proposed soluation set (x) of the original linear system.

The proof of the above mentioned theorem was given by Author in previous work (1,2). Then it is not need to give same proof again in this paper. The above mentioned theorem can be applied on the solution of normal equations of linear least squares problem as following; after application of theorem (Holotransformation) once we will get following auxiliary system and solution vector.

$$(A^T.A).(A^T.A)^T\tilde{X} = A^T b; \quad \tilde{X} = X^T(AT.A) \qquad (1)$$

After application of theorem two times we will get following auxiliary system and solution vector.

$$(A^T.A)(A^T.A)^T(A^T.A)(A^T.A)^T.\tilde{X} = A^T b,$$

$$X = \tilde{X}^T(A^T.A)(A^T.A)^T(A^T.A) \qquad (2)$$

AN APPLICATION OF GAS OIL'S EXPERIMENTAL DATA

Equation 3 is choosed model equation for forced convection where the proportionality constant and exponents must be evaluated from experimental data.

$$hD/k = \alpha(D.G/\mu)^p . (c\mu/k)^q \qquad (3)$$

The data are given in Table 1 which are obtained by Morris and Whitman on heating gas oil with steam in a 1/2 in. IPS pipe with a heated length of 10.125 ft (6).

$\alpha$, p and q can found algebraically by taking the data for three tests points or graphically and linear least squares approach which are preferable for the correlation of a large number of points.

Let's say;

A=h.D/k, B=DG/$\mu$, C=c$\mu$/k and Let's insert A,B,C into Equation 3

$$A = \alpha B^p C^q \qquad (4)$$

Taking the logarithms of both sides of Equation 4

$$1nA = 1n\,\alpha + p1nB + 1nC$$

which reduces on logarithmic coordinates to an equation of the $K_1$ = 1nA $K_2$ = 1nB, $K_3$ = 1nC, X = 1n$\alpha$

from $K_1 = X + pK_2 + qK_3$ (5)

Derivation of normal equation of (LLS) problem can be solved as following;

$$ö = \mathrm{Min} \sum_{i=1}^{n} (x + pK_2 + qK_3 - K_1)^2 \qquad (6)$$

$$\delta\phi/\delta x = 2\sum_{i=1}^{n}(x + pK_2 + qK_3 - K_1)(1) = 0$$

$$\delta\phi/\delta p = 2\sum_{i=1}^{n}(x + pK_2 + qK_3 - K_1)(K_2) = 0 \qquad (7)$$

$$\delta\phi/\delta q = 2\sum_{i=1}^{n}(x + pK_2 + qK_3 - K_1)(K_3) = 0$$

$$nX + \sum_{i=1}^{n} K_2 p + \sum_{i=1}^{n} K_3 q = \sum_{i=1}^{n} K_1$$

$$\sum_{i=1}^{n} K_2 x + \sum_{i=1}^{n} K_2^2 p + \sum_{i=1}^{n} K_2K_3 q = \sum_{i=1}^{n} K_1K_2 \qquad (8)$$

$$\sum_{i=1}^{n} K_3 x + \sum_{i=1}^{n} K_2K_3 p + \sum_{i=1}^{n} K_3^2 q = \sum_{i=1}^{n} K_1K_3$$

Calculated values are;

$\sum_{i=1}^{n} K_1 = 64.047562$, $\sum_{i=1}^{n} K_2 = 117.9828$,

$\sum_{i=1}^{n} K_3 = 47.41084083$, $\sum_{i=1}^{n} K_1K_2 =$ 589.34913, $\sum_{i=1}^{n} K_1K_3 =$ 232.3962496,

$\sum_{i=1}^{n} K_2K_3 =$ 429.025376, $\sum_{i=1}^{n} K_2^2 =$ 1079.3908, $\sum_{i=1}^{n} K_3^2 = 173.093272$

Derivated normal equation of test problem is as following;

$$\begin{aligned} 13X + 117.9828p + 47.4108q &= 64.0476 \\ 117.9828X + 1079.3908p + 429.0254q &= 589.34913 \quad (9) \\ 47.4108X + 429.0254p + 173.093q &= 232.39625 \end{aligned}$$

Above mentioned simultaneous linear equation (Normal equations of test problem) can be solved as following; Result is

q = -2.047, p = 0.6386, x = 6.5964, $\alpha$ = INV.1n (X) = 732.4536

The solution of normal equations of test problem can be proceed over the given theorem is the following steps.

1)

$$A = A^T = \begin{bmatrix} 13 & 117.9828 & 47.4108 \\ 117.9828 & 1079.3908 & 429.0254 \\ 47.4108 & 429.0254 & 173.093 \end{bmatrix} \quad (10)$$

2)

$$A.A^T = \begin{bmatrix} 16336.725 & 149223.76 & 59440.437 \\ 149223.76 & 1363067.2 & 542941.0 \\ 59440.437 & 542941.02 & 216271.76 \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} 16336.725 & 149223.76 & 59440.437 \\ 149223.76 & 1363067.2 & 542941.02 \\ 59440.437 & 542941.02 & 216271.76 \end{bmatrix} . \begin{bmatrix} \tilde{x} \\ \tilde{P} \\ \tilde{q} \end{bmatrix} = \begin{bmatrix} 64.0476 \\ 589.34913 \\ 232.39625 \end{bmatrix} \quad (12)$$

3) The solution of equation 12 by any method

$$\tilde{X} = -1.3543407, \tilde{p} = 0.19758, \tilde{q} = -0.12272$$

to get final solution;

$$\begin{bmatrix} -1.3543407 & 0.19758 & -0.12272 \end{bmatrix} = \begin{bmatrix} 13 & 117.9828 & 47.4108 \\ 117.9828 & 1079.3908 & 429.0254 \\ 47.4108 & 429.0254 & 173.093 \end{bmatrix}$$

$$\begin{bmatrix} -0.11363 & 0.827212 & -0.6854772 \end{bmatrix}$$

X=-0.11363, p=0.827212, q=-0.6854772, $\alpha$=INVIn (x)=0.892588

CONCLUSION AND DISUSSION

The fitted equations of gas oil's experimental data are as following by Kern, linear least squares (LLS) estimation and proposed Holotransformation

$$(h_iD/k)=0.0115\ (D.G./\mu)^{0.9}.(c\mu/k)^{1/3}\ \text{(Kern)} \quad (14)$$

$$(h_iD/k)=732.4536\ (D.G./\mu)^{0.6386}.(c\mu/k)^{-2.047}\ \text{(LLS estimation)} \quad (15)$$

$$(h^iD/k)=0.8926\ (D.G./\mu)^{0.8272}.(c\mu/k)^{-0.6855}\ \text{(Holotransformation)} \quad (16)$$

Table 1 is shown observed data and calculated values of Kern, LLS estimation and proposed Holotransformation for forced convection equation of gas oil (Figure 1).

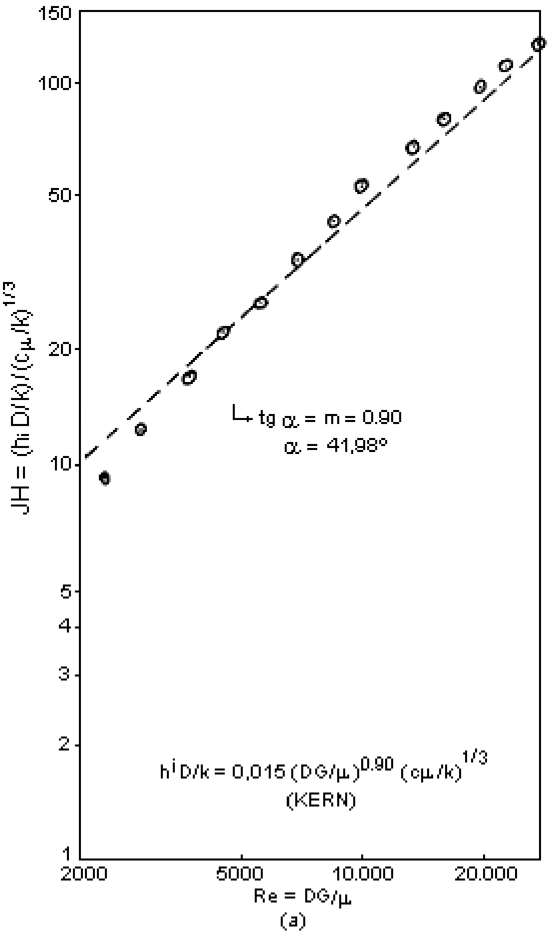
Two tests are often performed two determine the validity of fitted model. First the "multiple correlation coefficient" (R) may be calculated, $R^2$=(sum of squares due to regresion (SUMSR) / sum of squares corrected total (SUMST) where;

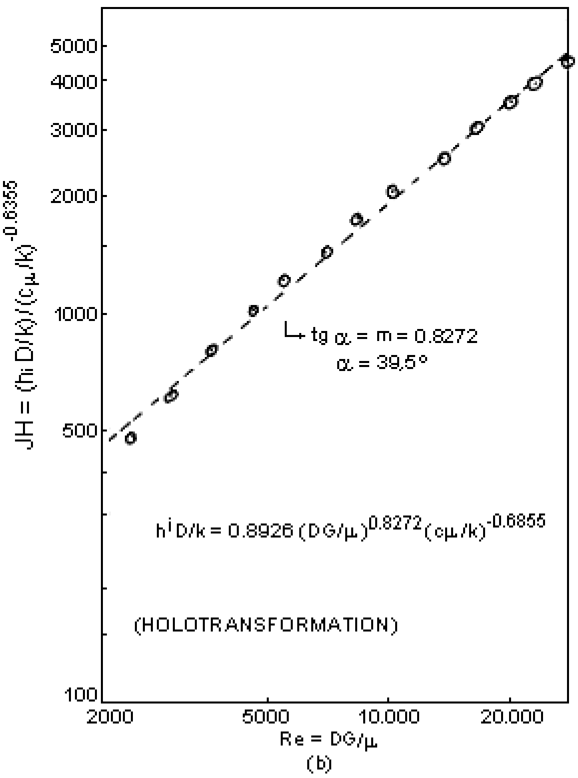
$$SUMSR = \sum_{i=1}^{n}(\hat{Y}_i - \overline{Y}_i)^2, SUMST = \sum_{i=1}^{n}(Y_i - Y_i)^2$$

$\hat{Y}_i$, is estimated values, $\overline{Y}_i$ is aithmetic mean of observed values $Y_i$. The value of (R) will be between 0 and 1 with R=1 corresponding to a perfect fit. Secondly, the least squares objective function ($\sum S_i^2$) is evaluated. For a perfect fit this value will be zero. On the other hand,

Table 1: Calculated values of Kern, (LLS) and proposed equations with observed data.

| DG/$\mu$ = b | c$\mu$ / k=C | ($h_1$D/k) observed = A | $A_{Kern}$ | $A_{LLS}$ | $A_{proposed}$ |
|---|---|---|---|---|---|
| 2280 | 47.2 | 35.5 | 43.734 | 38.247 | 38.0966 |
| 2825 | 46.7 | 46.3 | 52.85 | 44.823 | 45.816 |
| 3710 | 43.3 | 62.3 | 65.86 | 62.27 | 60.45 |
| 4620 | 41.4 | 79.5 | 79.04 | 78.527 | 74.74 |
| 5780 | 40.7 | 95.0 | 96.15322 | 93.8226 | 91.022053 |
| 7140 | 38.7 | 120.5 | 114.35711 | 119.04398 | 112.21739 |
| 8840 | 37.7 | 147.5 | 137.38884 | 143.9507 | 136.32647 |
| 10850 | 36.5 | 176.5 | 163.43642 | 175.30409 | 165.12492 |
| 14250 | 35.3 | 223.0 | 206.56446 | 223.41456 | 211.68641 |
| 17350 | 35.1 | 266.5 | 246.13 | 256.5 | 250.09 |
| 20950 | 34.1 | 313.0 | 288.85 | 306.71 | 298.15 |
| 25550 | 32.9 | 356.0 | 341.25 | 374.66 | 360.09 |
| 30000 | 32.7 | 407.0 | 393.51 | 420.0 | 413.00 |



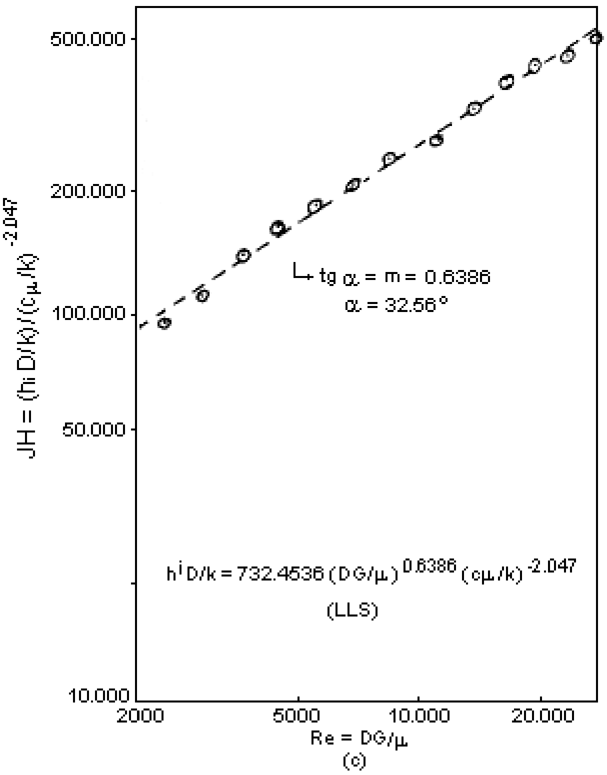




Figure 1: Plots of Re v.s. $J_H$ for KERN Eq., Holotransformation Eq. and (LLS) Eq.

the term "residual" ($\sum S_i^2$) is preferred and refers to uncertainties, not necessarily to errors (5,7).

The calculated ‘multiple correlation coefficient' (R) are 0.8237 for Kern, 1.219228 for (LLS) estimation and 1.0026 for the proposed Holotransformation.

The calculated ($\sum S_i^2$) are 2103.3731 for Kern, 685.14072 for (LLS) estimation and 1042.2586 for proposed Holotransformation.

The least squared estimation is minimizing the summation of least squares of residual vector and it produces the average. The proposed Holotransformation approach is minimizing the difference of standard deviations between observed and calculated values and it produces the Mode. It is well known that the Mode is more powerful statistics than the others such as average, midrange and median.

In the proposed approach the residuals of some data points are excellent according to Kern and (LLS) estimation. On the other hand the residuals of some data points are not good because of such as data points have some observation error which are arisen from determination of $h_i$. Finally, the proposed approach gives more optimal solution for experimental data.

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