A CONTRIBUTION TO THE ISLAMIC CHRONOLOGY

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It is well-known that the Moslems use a lunar calendar. Since the length of the lunar year is different from that of the solar year (the difference is about 11 days), one cannot transform the Moslem dates into the Gregorian ones (and vice versa) simply by adding or substracting a constant. There are, of course, some reduction tables, but I have nowhere met any simple analytical formula, which would be much more comfortable for practical use, e.g. for programming. On the other hand, such a formula is not difficult to derive.

For simplicity, we shall use the Scaliger's Julian date (JD), that can easily be converted into the date of the Gregorian calendar, or be deduced from it. Our task is then to find a relation between JD and the date of the Moslem calendar.

There are some differences between the calendar systems of various countries, but the basic features of this calendar are the same all over the Islamic world:

1. In the common years, the odd months have 30, the even ones 29 days, so that the year's length is 354 days.

2. In the leap years, the intercalary day is added to the 12th month, hence its length becomes 30 days and the year's length is then 355 days.

Let M be the month, D the day in month of the Moslem calendar. Then the number of days from beginning of the actual year to the current day (incl.) is

$$N = D + 29 (M-1) + (M/2) \qquad [1]$$

The most widely used system is the so-called Arabic cycle, the length of which is 30 years, i.e. 19 common and 11 leap ones. Particularly, the 2nd, 5th, 7th, 10th, 13th, 16th, 18th, 21st, 24th, 26th, and 29th year of the cycle are the leap years. As the cycle contains 11 intercalary days, its full length is 30x354+11=10631 days. Let H be the year of the Islamic era (the hegira); we can express it in the form

$$H = 30 h_1 + h_2 + 1, \qquad [2]$$

where h_1 = [(H-1) / 30] and h_2 is the remainder after this division. Then the leap years are those for which h_2 = 1, 4, 6, 9, 12, 15, 17, 20, 23, 25 or 28. Now, it is clear that gira to the end of the previous 30-years cycle, is

$$N_1 = 10631 h_1, \qquad [4]$$

2. The number of days, from beginning of the current cycle to the end of the previous year H-1, is

$$N_2 = 354 h_2 + Z (h_2), \qquad [5]$$

where Z (h_2) is the number of intercalary days (i.e. of leap years) from beginning of the cycle to the year H-1 (incl.),

The function $Z(h_2)$ obtains the following values:

h_2	z	h_2	z	h_2	z	h_2	z	h_2	z
0	0	6	2	12	4	18	7	24	9
1	0	7	3	13	5	19	7	25	9
2	1	8	3	14	5	20	7	26	10
3	1	9	3	15	5	21	8	27	10
4	1	10	4	16	6	22	8	28	10
5	2	11	4	17	6	23	8	29	11

It is not difficult to find an empirical formula

$$Z (h_2) = [(11h_2 + 14) / 30] , \qquad [6]$$

or which gives the same result,

$$Z (h_2) = [(7h_2 + 8) / 19], \qquad [7]$$

The value of N_2, evaluated from [5], are identical with those given by Ginzel (1). For the countries, where the 15th year of the cycle has been chosen leap instead of the 16th, the constant 14 in [6] can be replaced by 15 (resp. 8 in [7] by 9); then Z [15] = 6, while the other values remain unchanged.

The initial day of the hegira (i.e. D=1, M=1, H=1) is

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Friday, 16th July 622 A.D., and the corresponding JD value is 1948440 (at noon). Therefore the complete expression for JD is

$$JD = N + N_1 + N_2 + 1948439, \qquad [8]$$

where N, N_1 and N_2 are given by [1], [4] and [5]. Inverting [8], we can write

$$h_1 = [(JD - 1948440) / 10631], \qquad [9]$$

because the remainder

$$N + N_2 - 1 = q, \qquad [10]$$

satisfies the condition $0 \leq q \leq 10630$. Further, one must find the highest value of h_2, for which [5] yields $N_2 \leq q$ (e.g. by a programme loop). Then H can be evaluated from [2], and the day and month, in the same way, from the equations [1] and [10].

Sometimes it is not necessary to know the date exactly, but only the corresponding year of the solar calendar. We show a simple method how to find the year of the Julian calendar, in which a given hegira year begins. Let's take the case D=1, M=1, i. e. the Moslem New year, into consideration. From [8], using [6], follows:

$$JD = (10631 (H+3) / 30) + 1947023, \qquad [11]$$

on the other hand, it is not difficult to show that JD can be expressed in the form

$$JD = 365.25 (Y + \tau) + 1721058, \qquad [12]$$

where Y is the year of the Julian calendar, and $0 \leq \tau < 1$. Comparing of these two equations gives, after some manipulations,

$$Y = [4h / 146] - 5872, \qquad [13]$$

with,

$$h = [10631 (H+6693) / 30] \qquad [14]$$

with the Turkish cycle, the situation is similar. It lasts 8 lunar year, from which 3, namely the 2nd, 5th and 7th, are the leap years, so that the cycle's length is 8*354 + 3 = 2835 days. The corresponding formulae obtain the following form:

$$h_{1'} = [(H-1) / 8], \qquad [3']$$
$$H = 8h_{1'} + h_{2'} + 1, \qquad [2']$$

($h_{2'}$ is the remainder in [3']),

$$N_{1'} = 2835\ h_{1'}, \qquad [4']$$
$$N_{2'} = 354\ h_{2'} + Z'\ (h_{2'}), \qquad [5']$$
$$Z'\ (h_{2'}) = [\ 3\ (h_{1'} + 1\ )\ /\ 8\ ], \qquad [6']$$
$$JD = N + N_{1'} + N_{2'} + 1948439, \qquad [8']$$
$$h_{1'} = [(JD - 1948440) / 2835\ ], \qquad [9']$$
$$N + N_{2'} - 1 = q', \qquad [10']$$

(q' is the remainder in [9']),

$$h' = [10631\ (H + 216) / 30], \qquad [14']$$
$$Y = [4^*\ h' / 1461] + 412, \qquad [13']$$

REFERENCE

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